Optimal Shape Design of Mechanical Components for Single and Multiple Objectives using Genetic Algorithms

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Certificate

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This is to certify that the work contained in the thesis entitled *Optimal Shape Design of Mechanical Components for Single and Multiple Objectives using Genetic Algorithms* by *Tushar Goel* is carried out under my supervision. This work has not been submitted elsewhere for a degree.

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Abstract

Finding the optimal shapes of the mechanical components is one of the most visited field of the optimization and also this is a difficult task The main difficulty lies in the method of representation of shape in an optimization algorithm Most optimal shape design procedures involve pre-fixing the shapes particularly a mathematical form (say through cubic splines or some polynomials) The task of the optimization method is the optimization of various parameters governing the mathematical form or identifying the optimal value of the co-efficients defining the mathematical functions Although in some cases when the problem is simple this method works but it cannot be generalised to find the shape of any arbitrary shape optimization problem The problems, where the information about the shape is not known before hand, finding the solution becomes very difficult

To overcome this difficulty, the representation scheme where the shapes are given by either the presence or the absence of the small material pieces, is used This way, the material shape can be represented by a string of binary numbers, where each binary variable decodes to the presence or absence of the material at a specified place A smoothing algorithm is used to remove the difficulties like sharp corners and point contact etc The shapes are analysed by using the finite element analysis to find out the stresses and strains Constant strain triangles are used as the basic elements for finite element analysis A hybrid approach based on the Genetic Algorithms (GA) is used to find out the optimal shapes For the single objective problems the objective is to minimize the weight The optimal solution obtained by the GA runs, undergoes a hill climbing local search, which ensures the convergence of the solution to the globally optimal solution

When the problem is solved for more than one objectives, it becomes a multi-objective optimization problem Here the objective functions are the minimization of the weight and the minimization of the deflection, which are conflicting in nature Then, a set of Pareto-optimal solutions is obtained as the solution to this problem, as one single solution cannot be the optimal solution To solve these types of problems, a number of multi-objective genetic algorithms (MOGA) are given in literature A specific multi-objective genetic algorithm, an elitist non-dominated sorting genetic algorithm (NSGA-II) is used Each solution obtained by the NSGA-II search undergoes the local search to give an optimal solution A weighted sum strategy is used to convert the multiple objectives into a single objective The weight vector is calculated by using the fitness values of the solutions in an adaptive way Two strategies are adopted for the weight calculation– one the fixed weight strategy, where weights are calculated on the basis of initial fitness values, and the second is the continuously updated strategy, where the weights are found by the continuous updating of the fitness values The number of the solutions obtained after

the hybrid approach may be too large for the designer to take into consideration A clustering approach is used to reduce the number of solution to the desired number

A number of test problems like the design of cantilever plate, simple supported plate hoister, etc are solved Both the single objective as well as multi-objective cases are handled Each problem is solved twice first -when the weight of the plate itself is not considered for the calculation of the load and second is the case when the load of the designed plate is also accounted for while calculating the load The results obtained by the application of hybrid approach to all these problems give us very interesting results In a number of cases the results obtained are very obvious and some times some not easy to visualize solutions are found to be optimal This makes the approach very efficient in handling the shape optimization problems Most interesting application of this hybrid approach is the design of the bicycle frame where a number of different bicycle frame shapes are obtained in just one simulation run

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Chapter 1

Introduction

1.1 Introduction

Design of the shapes of the mechanical components is not a new activity In fact, the shape optimization is one of the most visited areas of the optimization The most popular method of shape optimization are the use of classical methods Since many classical methods require the function and the derivative information Therefore, the shape of the component is decided to be approximated by a mathematical function mostly a polynomial in nature, as the mathematical functions can be easily handled by the classical methods The co-efficients of the polynomial and other parameters governing the mathematical function considered are obtained using the different optimization methods, to find the optimal shapes [2 11 12 16] This methods suffer from the disadvantage of requiring the knowledge of the shape before hand In any arbitrary problem where the loading and support conditions are such that it is difficult to guess the shape, the method finds it difficult to get the shape Besides this, the problems where the properties of the material under consideration are not constant over the search domain, the initial guess of the shape is not at all easy The shapes represented by the mathematical functions also find it difficult to find the holes in the shapes, which may be helpful in some of the cases as the material can be sometimes reduces from the areas of low stress Despite these drawbacks, the classical methods are most popular due to the absence of any other approach which does not suffer from these drawbacks

Here in this work a representation scheme other than the representation of the shapes by the mathematical functions is adopted Here the shapes are derived from the rectangular plate, divided into a number of small elements The shapes are represented by the presence or absence of the material elements [4, 5, 13] Thus each shape is presented by a string of binary numbers Each 1 in the binary string signifies the presence of the material and 0 corresponds to the void in that position The basic shape is extracted from this string, through a smoothing technique, eliminating the string representing the unconnected regions FEM analysis is used to find out the stresses and strains for these shapes A hybrid approach based on the genetic algorithms (GAs) is used to solve the real world problems of

the shape optimization Binary GA is used to find the optimal shape of the component and then a hill climbing local search is used to solve the problems

The problems can be of optimization of single objective or of optimization of multiple objectives with a few constraints like geometry constraint and stress and strain constraints For the multi-objective optimization problems, the solution is not a single point instead it is the set of solutions which are trade-off solutions known as the Pareto-optimal solutions To find the Pareto-optimal set a number of multi-objective GAs (MOGA) are suggested [6, 15 21 24 29 32, 38] A specific MOGA- elitist non dominated genetic algorithm (NSGA-II) is used to find the Pareto-optimal solution set Each solution of the solution set obtained from the MOGA simulations are then improved by the local search The use of the local search ensures a better convergence to the global Pareto-optimal front and also help in reducing the size of the non dominated solutions set to a reasonable number Finally a clustering algorithm is used to find the desired number of different solutions The efficacy of the proposed method is shown through a number of test problems like—the design of the cantilever plate design of simply supported plate, design of a hoister, etc The results obtained are very interesting as the approach gives some very intuitive results and also it is able to find results which are not so easy to visualize The proposed method gives the solutions that show the trend of the placing of the material in the optimal manner Since the solutions obtained are evolved through the GA which mimic the nature, the solutions obtained are very near to the global optimal solution And the same is shown with the help of a number of test problems

1.2 Previous work in the field

In the activities related to design of the mechanical components, shape design is the first step [18] Since the development of the numerical techniques during the world war-II, shape design of mechanical components has also become very popular [2, 11, 12, 31] Most of the studies are classified into two major categories –

1 Shapes are represented as a mathematical function

2 Shapes are formed by deleting material from an initially chosen shape

In the first approach, the design of the shapes is done by pre-deciding a mathematical function and then finding optimal parameter of shapes [16, 33, 35] This approach requires the knowledge of the shapes as well as the proper range of the variables is to be decided Continuity and compatibility conditions are to be satisfied on the boundary points The second approach is more elegant and used more sparingly [14] The rectangular plate is taken as the initial shape and a number of elementary shapes such as circle,

square, or rectangle etc are deleted from different places The optimization problem is to find the optimum place and the size of these elementary shapes This approach is very simple in principle but suffers from a number of problems Often the shapes obtained by the subtraction can not be realized The discontinuities in the boundary shapes are present Since a number of elementary shapes form the boundary of the resulting components, decision variables are highly inter-dependent

A number of researchers [4, 5, 13, 19, 22, 23, 28] have shown that the shapes can be represented by the binary material/void representation The use of genetic algorithm to this type of shape optimization problems gives very interesting results A number of studies are done for the single objective optimization problems [4 5, 13 19, 22] The researchers have used the Finite Element Analysis to analyze the solution and find the stress and strains The representation taken by the researchers suffers the drawback of not considering the cases of point contacts, which may lead to very high stress Then the results of the study may not remain the optimal The use of the genetic algorithm approach to solve some real life problems is also given in [28] where the design of a car bumper is carried out using the genetic algorithms The use of single objective GA to solve the bicycle frame design problem is carried out in the [23]

But so far nobody has carried out the design of shape optimization problems for the cases when more than one objectives are considered This study presents the design solution with the new representation which does not have the problem of sharp corners and the point contact for single and multiple objective problems And the solutions are solved through a hybrid approach combining the advantage of the GA and hill climbing local search

1.3 Organization

This study is presented in all six chapters The brief description of all the chapters is helpful in studying the work in an organized manner This is given here –

Chapter 1 is the introduction to the work In this chapter, the difficulties of the classical approaches in solving the shape optimization and the brief outline of the proposed approach is discussed The work so far done by others is also discussed briefly

Chapter 2 discusses the method how GA can be applied to the problem of shape design This chapter also discusses the details of the FEM analysis, the 2-D crossover operator, and the method of local search for the single objective problems

Chapter 3 presents the description of the different test problems taken into consideration Then the

results for the single objective optimization are presented In this chapter the results of comparison of the local search method and the suggested hybrid method is also presented

Chapter 4 bears the brief introduction to the terminology used in the multi-objective optimization problems and then briefly NSGA-II algorithm is also presented The local search method and different weight calculation strategies are discussed

Chapter 5 presents the results for the test problems when the number of objectives is more than one The resultant Pareto-optimal set and the shapes obtained are presented In the end of the chapter the results for the design of bicycle frame is presented, which are found to be the most exciting

Chapter 6 summarizes the hybrid method, main results and the conclusions drawn from the results The chapter also gives some of the future scope of the work, which will be useful for interested readers

1.4 Closure

The chapter gives an overview of the study This discusses the drawbacks of the classical methods used for solving the shape design problems The scheme of representing the shapes by the binary strings is given in brief and then outlines of the proposed hybrid method to solve the shape design problems are sketched for both the cases of single objective problems and the multi- objective problems The previous work done is mostly done for optimizing the shapes formed by the group of rectangular elements, without any consideration to the manufacturing problems Most of the work is done for the single objective optimization Some interesting practical applications are also reported in history The organization of the report helps in guiding the reader to the section of his/her interest

Chapter 2

Genetic Algorithms for single-objective optimization

The Genetic Algorithms (GAs) is a reliable approach of solving the optimization problem It mimics the natural evolution process to find the optima [20] It has the potential of finding the optimal solution which may be surrounded by a lot of local optima or it may be highly multi-modal It can be used for other purposes as well, like design of components, scheduling routing etc Here the discussion is centered around the question—How GAs can be applied to the shape optimization problems or the problems of structures which seem to have no relation with the domain of genetic algorithms Answer to this question tells an approach how the problems from different walks of life, can be modelled to apply GA This chapter mainly deals with the same aspect

2.1 Standard genetic algorithm

Genetic Algorithm(GA) is the optimization technique, which mimics the nature in order to find the global optima of a given problem This is inspired from the concept of *"Survival of Fittest"* It means that only those individuals, which are better than others in one or other aspect survive Unlike classical optimization methods GA deal with a population of individuals In Binary GA individuals are represented by the binary strings called chromosomes These chromosomes are the string of 0's and 1's called *allele* In real coded GA, chromosomes comprise of string of real numbers Salient features of a standard simple GA are as following

1 Selection

2 Crossover

3 Mutation

2.1.1 Selection

For a given population of individuals only those individuals are selected which are better in fitness than others. This way, GA follows the Darwinian law of survival of the fittest and by this selection our search is guided towards the optima. More importantly, this method does not reject the solution which is not the best. This solution may have the information, useful in finding the global best solution. Most used methods for selection are

1. Tournament selection
2. Stochastic universal sampling
3. Roulette wheel selection

2.1.2 Crossover

In crossover two individuals mate and produce children. This is a very important operator which create the new solutions, by using the properties of the parents. This creates the solutions in the region other than the existing region. This explores the search space to find the best region. It is supposed that the individuals which are surviving, have some good string, which let them alive. So good parents combine to produce better children. Two types of crossovers are mostly used

1. Binary crossover
   Deals with chromosomes with binary numbers
2. Real crossover
   Deals with chromosomes with real numbers

2.1.3 Mutation

This mimics the sudden changes occurring in the nature, that creates the individuals, different from the individuals of its own category. This operator is particularly helpful when the search has stuck at some local optima or search is going slowly. This can also create solutions which are not good. They will be taken care by the selection and other operators.

The algorithm of simple GA is shown in Figure 2.1

2.2 Representation scheme

For all the different problems a rectangular plate is taken as the basic shape. The plate material has the physical significance whereas the representation in form of the 0's and 1's do not have any physical

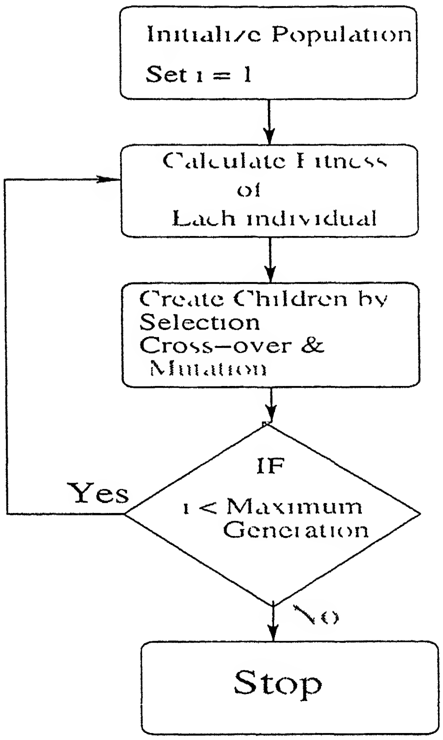

Figure 2 1 Simple genetic algorithm

significance but these strings of 0's and 1's can be processed by binary GAs Thus, here is a discrepancy in the two representations This is avoided by finding the compatibility between these representations For this purpose the rectangular plate is divided in small regular pieces which are called *elements* These elements are our basic *building blocks* Each element can either be present or absent If the element material is present a value '1' is assigned to it and if element is void it is specified as '0' This way, material is presented in the form of a sequence of 0's and 1's These shapes in form of the string are processed by the GA's Since the body taken is two dimensional so this string is converted in the two dimensional array by filling the array positions by the string members sequentially from the left as shown in the Figure 2 2

This is more clear when a material shape is represented in the form of an array and this array is converted into a binary one-dimensional string and vice-versa is also true

Example: The representation scheme is more clear by the example Here is a randomly created string of length 100

1111000111 0100010111 1001101000 0011110010 1110000011

1100100000 1010101000 0000100110 0011101110 0001010000

Two dimensional representation

The corresponding two-dimensional array of size 10×10 is given in the Figure 2 3

1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20

Figure 2 2 Representation scheme

1	1	1	1	0	0	0	1	1	1
0	1	0	0	0	1	0	1	1	1
1	0	0	1	1	0	1	0	0	0
0	0	1	1	1	1	0	0	1	0
1	1	1	0	0	0	0	0	1	1
1	1	0	0	1	0	0	0	0	0
1	0	1	0	1	0	1	0	0	0
0	0	0	0	1	0	0	1	1	0
0	0	1	1	1	0	1	1	1	0
0	0	0	1	0	1	0	0	0	0

Figure 2 3 Array representation

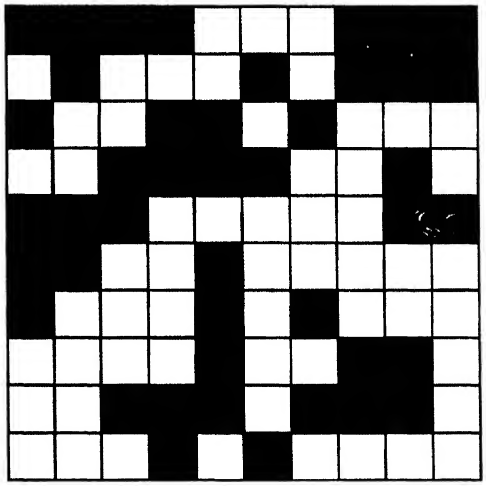

Figure 2 4 Material representation of the string

Material Representation

The material representation for this 2-D array is given in the Figure 2 4 If the material shape is given, one can find what will be the corresponding string and if a string is given the material presentation for the corresponding string can be easily obtained

2 2 1 Connectivity and smoothing

Converting the binary string into the material shape, gives a skeleton of the shape but this shape is not meaningful unless it is connected Unconnected region will not be able to take any load and hence without contributing any thing significant, it will increase the weight Besides this it will not be accounted while producing the object physically In order to find the connectivity the biggest connected region (cluster) is found To find the largest connected region connectivity between elements is defined using following definition

Definition 1 *To define connectivity two terms neighbour and group are defined as following*

1 **Neighbour.** *The material elements which are sharing at least one corner are called neighbours*

2 **Group** *The collection of all the material elements, connected to each other by means of neighbours constitute a group The group is often called as* **cluster**

The most important property of the group is that if one starts from any of the element of the group one can reach any other element of the same group

2 2 2 Steps for finding connectivity

Connectivity for any element is obtained in following steps

- One single element for which, material is present and which does not have any neighbour, is termed as a separate group
- The material element, which have at least one neighbouring material element, belongs to the common group same as the neighbour and the group number is smaller of the two group numbers
- Classify all the elements as the members of the different groups or form different groups, such that each element is *exactly* the member of *one* group *only*
- The group which has maximum numbers of elements among all these groups, is found and that is taken as our basic shape for that string i e the string gives the shape as identified by the biggest group

The biggest cluster is taken as the unsmoothed shape and the string is now repaired by substituting all the elements which do not contribute to the biggest cluster by '0' Thus lamarckian approach followed here [25] The shape after clustering for the string taken for the example is shown in the Figure 2 6

2.2.3 Smoothing the component

After extracting the basic shape from the string a region, connected by the edges or have corner contacts i.e. the two elements are connected by just sharing one corner, may be obtained. These types of configuration are feasible for theoretical works, but for designing of the object under action of actual load is not possible as the design will be highly unsafe at the point, where the region is just sharing one node only because there is not enough material to support the load and hence the stress may exceed the allowed limit. So with this representation scheme the shapes with crossed-ribs sort of arrangements which are often found to be useful in a number of cases cannot be found.

In order to overcome this problem, smoothing operations are applied. Smoothing is done by following the given conditions. One typical case of each smoothing method is shown in Figure 2.5.

Condition 1: When there is a hole made by the single element, it is filled up by replacing the void by a material element.

Condition 2: When a void is having two material elements adjacent to it, it forms a sharp corner and this will lead to high stresses. To avoid it a right angled triangle is put there such that the sharp corner gets eliminated. This filling of the triangle is in line with the concept of placing the fillet near the sharp corner.

The use of triangles can be justified by the fact that the fillet can be modelled as a polynomial of high degree where as the triangle can be taken as a fillet with a polynomial of degree one.

Condition 3: When the void element is cordoned by three material elements, the triangles are placed such that $3/4th$ of the void is filled. Which $1/4th$ part is to remain void is decided by the side of the vacant element. If right hand element is absent, the 1/4th portion of the right side can be left as void.

Condition 4: When the arrangements like shown in first part of the Figure 2.5 (sharing of one node only) are encountered, two right angled triangles are placed such that, a crossed rib like shape is obtained.

After smoothing the final shape represented by a string is obtained. This shape becomes a candidate for the consideration of finite element analysis, provided it satisfies the geometry constraints, if any. The final shape represented by the string taken in example is shown in Figure 2.7.

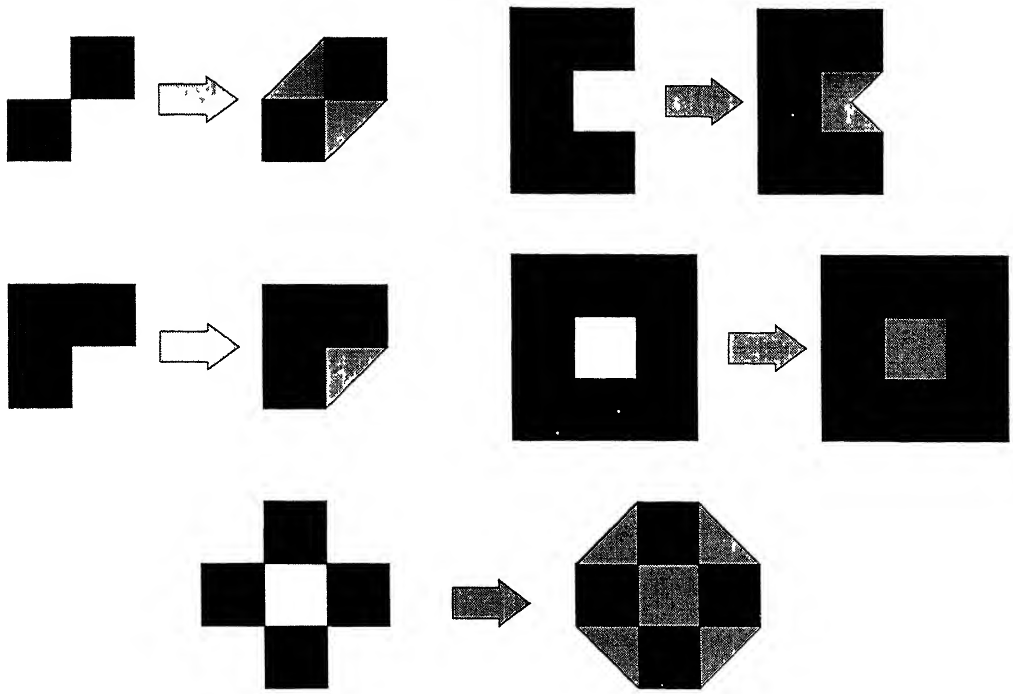

Figure 2 5 Different smoothing operations

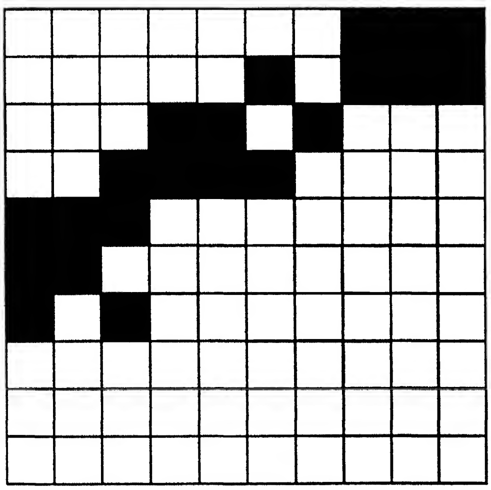

Figure 2 6 Biggest cluster

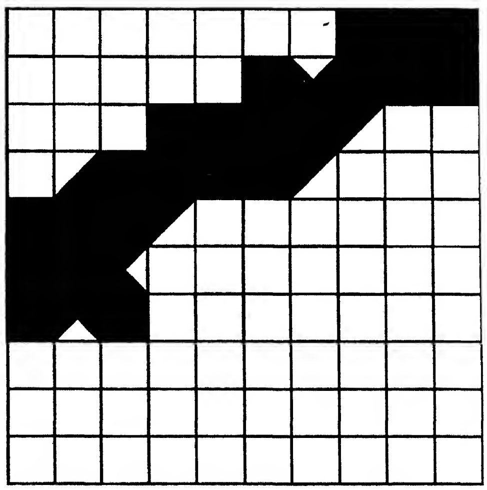

Figure 2 7 Smoothed shape

2.3 Single objective problems

Single objective problems are the most basic optimization problems Most of the classical methods are applicable to single objective problems Many classical methods for multiple objectives convert the problem into a single objective problem through a number of means and then solve it. So it is essential

to find whether this strategy of finding the shapes using Genetic Algorithms based hybrid method works well or not The problem is formulated with an objective of minimization of the weight Constraints which all the feasible solutions must satisfy, are imposed First constraint is that the structure must not violate the stress constraint and second, the maximum deflection must be less than a allowed level

2 3 1 Objective function

For Single objective problems the objective function is the weight of the object, in most of the cases

$$Weight = Area * Thickness * Density \tag{2 1}$$

For all the practical designs the thickness and the density of the material remains constant for all the individuals and over the generations So the area of the individual shape can be used as a measure of the weight Thus the problem requires the weight or the area of the shape to be minimized

2 3 2 Constraints

The shape used for carrying the load, must not fail while doing so To avoid failure, the limits on certain parameters are set and these constitute constraints Following constraints, are taken into consideration

- **Geometry constraint** Any structure is feasible, only if it satisfies the geometry constraints The shape under consideration must have the nodes for applying the load and also it must have the essential number of support nodes If any of these conditions is not satisfied the shape is rejected and a high error value is assigned

- **Stress constraint** Any structure is safe, so far, the stresses arising due to the loading does not exceed the maximum stress, the material can sustain The *maximum stress* arising at any point must be less than the *allowed stress*

$$[\sigma_{max}] \leq [\sigma_{allowed}] \tag{2 2}$$

  In most of the cases *allowed stress* is same as *yield stress* Then

$$[\sigma_{max}] \leq [\sigma_{yield}] \tag{2 3}$$

- **Displacement constraint:** This constraint is important as the design of shapes is considered It has been observed that some structures are safe from the stress point of view but they fail due to excessive deflection So it is necessary to keep the maximum deflection in the structure under a specified limit to avoid it from failing due to excessive deflection This is given as

$$[\delta_{max}] \leq [\delta_{allowed}] \tag{2 4}$$

2 3 3 Material selection

The properties of the material over the elements can be controlled, making it easy to handle *different metals* or the *variation* in the properties This is an advantage of using the small element concept Thus the design of the optimal shapes when the properties of the material are varying over the domain can be carried out easily, which is otherwise a very difficult job to design By this method different loading conditions like distributed load, axial load or vertical load etc can also be handled easily

2 4 Fitness and error evaluation

Genetic algorithm works with a population of individuals Each individual is assigned a fitness according to its function value(s) GA tries to maximize the fitness Since this is a constrained optimization problem so for each individual according to its constraint violation the error value is assigned The error is decided by finding the geometry constraint and the stress and the displacement constraints

2 4 1 Geometric constraints

The shape obtained after smoothing is not always feasible If the individual does not have the node where the load is applied, the material will be unstressed, and the finite element analysis to find the stress or displacement will have no meaning If the individual does not have the support nodes, the material under the action of loads will move Then the problem will be of the dynamics which is not solved here In these conditions the geometry constraint is infeasible and the individual is assigned a large error The individual is feasible only if all the nodes where body is supported and the nodes where load is applied are present

2.4 2 Stress and displacement evaluation

The stresses and displacements are required for the evaluation of the constraints Finite element analysis is used for evaluation of stresses and displacements The details of finite element analysis, are given in separate section The maximum stress and maximum strain values at any point appearing in the shape under consideration, are checked to find the violation of the constraints

2.4 3 Fitness evaluation

The objective is to minimize the weight More is the weight, less is the fitness Since the density of the material and the thickness of the plate are taken constant over the plate, the area of the shape can be directly taken as a measure of weight The fitness is calculated by finding the plate area of the shape

$$Fitness = \text{Maximum plate area} - \text{Area of individual shape}$$

GA will maximize the fitness

2.4.4 Constraint handling

The steps for constraints handling are given as following

1. Scaling the stress and displacement constraints

2. Find the square sum of the constraint violation (if any). If none of the constraints is violated corresponding violation is set to zero. Mathematically the representation is as following

$$CV_1 = \begin{cases} \frac{|\sigma_{max}| - \sigma_{allowed}}{\sigma_{allowed}} & \text{if } |\sigma_{max}| > |\sigma_{allowed}| \\ \text{NULL} & \text{else} \end{cases} \tag{2.5}$$

$$CV_2 = \begin{cases} \frac{\delta_{max} > \delta_{allowed}}{\delta_{allowed}} & \text{if } \delta_{max} > \delta_{allowed} \\ \text{NULL} & \text{else} \end{cases} \tag{2.6}$$

where

$$CV_i = i^{th} \text{ Constraint Violation}$$
$$CV = \sum_{i=1}^{k} CV_i$$

where

$$k = \text{number of constraints}$$

3. Tournament selection is done to handle the constraints as suggested by Deb, 1999, [9]. Tournament size taken is two. Winner of the tournament between two solutions is decided by following the constraint domination conditions

Condition 1: If any of the individual is feasible and other is not, select the feasible individual

Condition 2. If both individual are infeasible select the individual with least error or constraint violation

Condition 3: If both individuals are feasible or both are infeasible with same constraint violation, select the individual with better fitness

This is shown elsewhere [9] that this strategy can handle the constraints efficiently

2.5 Types of operators used

For different operations of genetics following operators are used

- **Tournament Selection** is used for selection. Tournament size of two is set
- **Binary 2-D Crossover** is devised and used. The details of this crossover are given in separate section
- Simple random **mutation** operator is used here

2.6 Hybrid method for shape optimization

GA's deal with the population of individuals. It has the potential to reach the global optimal solution but for this the population size must be sufficient [17]. In these problems each feasible individual has to undergo the finite element analysis, which is computationally very costly. So the population size is taken small to reduce the computation time. The reduced population size may cause convergence to the local optimal solution in a limited number of generations or it may prolong the search. To reduce the time to get the best solution and to ensure the convergence of the solution to global optimal solution the knowledge of schema processing by GA's is exploited. Schema processing means that good strings come closer to find the optimal solution. GA's do schema processing to bring the short good schema's together

The classical methods, work very well when the search space has the unimodal optima or if the solution starts with a very good initial solution. This will not work very good and has a large probability of getting stuck in the local optima, if the search space has too many optima. To ensure the convergence to the global optima both of these concepts are applied here. Initially, the GA's start with the randomly generated population and process it to get the good arrangement of the strings. This takes the search to some optimal solution, which may not be the global optima. Now, the hill climbing local search method similar to the steepest descent method is applied, with the solution obtained from the GA as the initial solution. This way it starts with a good initial solution and takes the search to the globally optimal solution quickly. The combined strategy is given following

1. Genetic algorithm is used to find the best solution.
2. Further refinement using one bit hill climbing strategy. This hill climbing is a local search method

The combined strategy has yielded very good results and proposed as a generic method of finding the global optimum method for the shape optimization problems

Hill Climbing Strategy

Step 1 Start from the left end of the binary string representing the domain and replace the muta the current bit i e replace '0' by '1' and '1' by a '0'

Step 2 Find out the biggest cluster and find connectivity

Step 3 If the solution satisfies the geometry constraint, calculate stress and displacement by finite element analysis Find the weight of the solution

Step 4 If the weight is improved and all the constraints are satisfied accept the change else restore the change and goto Step 1 unless the end of the string is not reached

2.7 Finite element modelling (FEM)

The **finite element modelling** is the most important and computationally costly part of this whole design process The finite element analysis is carried out on the shape obtained by the decoding of the string The finite element analysis gives the displacements as the primary variables These results are post processed to find the stresses developed in the body and the maximum stress developed in the body The maximum displacement appearing in the body is also extracted from the data Here

- Maximum stress is obtained by using the *Von-Mises Criterion*
- Displacement is obtained by finding the vector sum of the displacements in the *x and y* directions at each node Maximum displacement is the maximum of these displacements
- The material is assumed to be *ductile*
- Constant strain triangle is used as the basic element for the finite element modelling

2.7.1 Steps in finte element model formulation

The finite element analysis is based on the *Principle of Virtual Work* This states
If a general structure, which is in equilibrium with its applied forces, is subjected to a set of small, compatible virtual displacements, the virtual work done by the external forces is equal to the virtual strain energy of internal stresses
Mathematically it can be given as

$$\delta U_e = \delta W_e \tag{2.7}$$

where

δU_e	virtual strain energy
δW_e	virtual work done by external forces

For the **finite element analysis** the steps given below are followed

Element formulation

The two dimensional region shown in Figure 2 8 is divided into straight-sided triangles For this each interior material element is divided in two triangles and those rectangular elements, which are on the boundary (including the boundary of the hole) are divided into four triangular elements as shown in Figure 2 9 The triangles are also divided into smaller triangular elements Nodes are placed on all the vertex points

The size of triangles need not be same The boundary can be more accurately modelled by putting

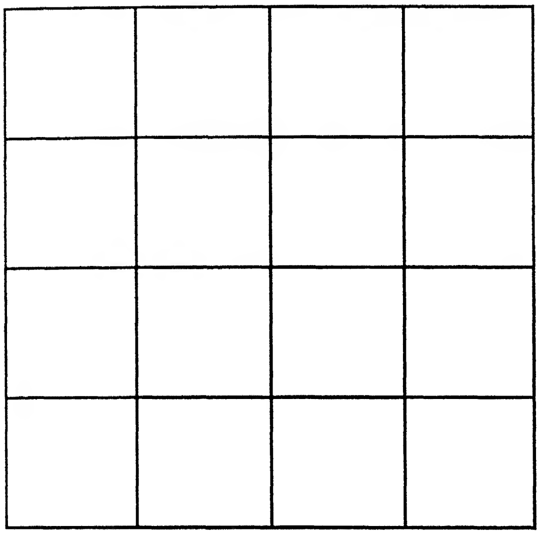

Figure 2 8 Rectangular domain

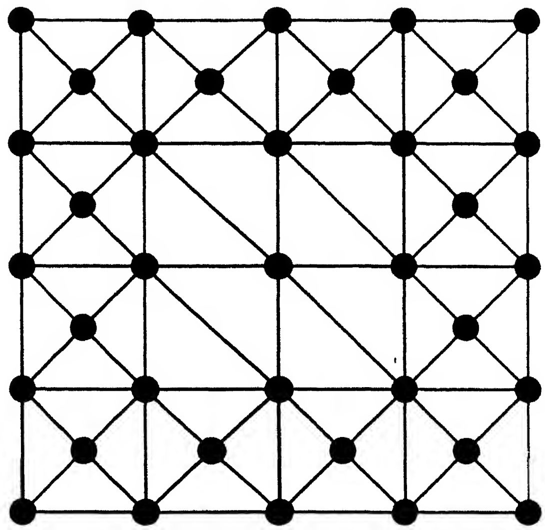

Figure 2 9 Meshed domain

more elements or a refined mesh there The elements of the different sizes are present in the same mesh

Shape function and degrees of freedom

Constant strain triangle elements are used for finite element modelling For this the shape functions are linear over the element The iso-parametric representation and shape functions in natural co-ordinates are used This gives the freedom to use different sized elements while finite element analysis

- One *dof* is assigned to displacement in each direction
- This is a two dimensional problem so each node has *two degrees of freedom* One dof is assigned to the displacement in each direction

Global node numbering and connectivity matrix

After creating the elements the global node numbering is done of all the nodes Global node numbering is done by tracing all the nodes from the top left corner to right bottom corner as shown in Figure 2 10

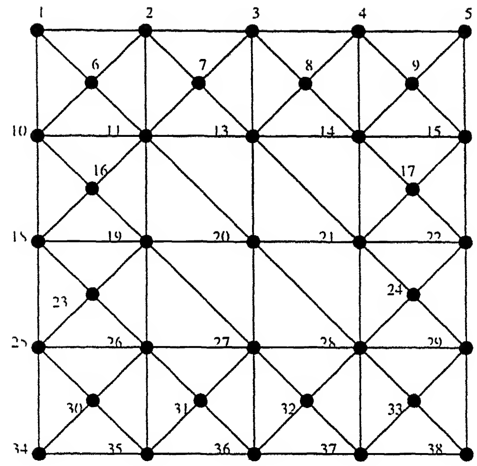

Figure 2 10 Global node numbering

After node numbering is done, connectivity is sought i e finding which element is attached to which element For this purpose moving from the top left material element, connectivity is found and accordingly the *Connectivity Matrix* is formulated

Assigning the geometric and material properties

The properties of the domain are assigned to each element For geometric properties, the co-ordinates information is stored and for material properties Young's modulus and density can be assigned to each individual element This helps in designing, when the search domain has more than one elements or the materials have variation in properties over the domain

Assembly

The differential equations are formulated and solved to get the matrix form By this *local element stiffness matrix* and similarly *local element force vector* is also obtained Assembly of the local stiffness matrices and force vector is done using the connectivity matrix This gives the *global stiffness matrix* and *global force vector*

Apply boundary conditions and solve the simultaneous equations

Boundary conditions on the global stiffness matrix are applied To apply the boundary conditions the diagonal element of the stiffness matrix corresponding to the degree of freedom is placed as 1 and corresponding value of the primary variable is substituted in the force vector Rest all the terms in the same row and column of that degree of freedom are set to 0 *Gauss Elimination Scheme* is used for solving the set of simultaneous equations This gives the value of the primary variables at all the nodes

Post processing

After obtaining the displacements, *post processing* is done to get the stresses in each element For this the *Stress-Strain Relations* are invoked and the *stress tensor* at each element is calculated By this stress tensor *principal stresses* are calculated To find the max stress at any point the *Maximum Strain Energy Theory (Von-Mises Failure Criteria)* is applied This gives the von-mises stress at each element Total displacement at any node is obtained by getting the vector sum of displacements in different directions at any node

- Maximum stress over the element is the maximum Von-Mises stress developed in the body

- Maximum displacement over the element is the maximum of the total displacements occurring at each node in the body

2 8 Binary 2-D crossover

The material domain is represented as the two dimensional domain So the crossover which operates on two dimensional strings is formulated This crossover respects the shape of the parents, whereas the normal one dimensional crossover does not guarantee to do so, hence found less effective for solving the problem The algorithm is given in Figure 2 11 The step-wise details are given as following

Step 1 Select two individuals Generate a random number and check whether the crossover is to be done or not, based on the crossover probability

Step 2 If crossover is to be done flip a coin and make a decision whether to go for a *row crossover* or *column crossover*

Step 3a If row crossover is to be done, for each row generate a random number and check whether random number is less than P_{cross}/No of rows If yes, swap the rows else not

Step 3b If column crossover is to be done, for each column generate a random number and check whether random number is less than P_{cross}/ No of columns If yes, swap the columns else not

Thus this crossover on an average swaps at least one row or one column in the two dimensional array The working of this crossover will be more clear with an example

For crossover two parents are chosen Let the first parent be P1 and second be P2 These two have to be crossed with a probability P_C of 0 800

	1	2	3	4
1	1	1	1	1
2	0	1	1	0
3	1	1	1	0
4	1	0	0	1

Parent 1 P1

	1	2	3	4
1	1	0	1	1
2	1	0	0	1
3	0	1	0	1
4	0	1	1	0

Parent 2 P2

The 16 bits parents when the shape is represented as a two dimensional array of size 4×4 are shown here. The first row shows the column number and the first column shows the row number. The stepwise procedure is given here.

Step 1 Generate a random number R. Let $R = 0.678$. Since $R < P_c$, hence, the parents are selected for crossover.

Step 2 $R = 0.185$. Since $R < 0.5$ so row crossover is performed.

Step 3 $P_r = P_c/N_r = 0.800/4 = 0.200$

For row 1 $R = 0.238$. Since $R > P_r$ so this row is not swapped.

For row 2 $R = 0.872$. Again $R > P_r$ so this row is not swapped.

For row 3 $R = 0.156$. Now $R < P_r$ so this row is swapped.

For row 4 $R = 0.333$. Since $R > P_r$ so this row is not swapped.

Step 4 Crossover for this pair of parents is complete.

After the crossover the children C1 and C2 are obtained. These are shown as following.

	1	2	3	4
1	1	1	1	1
2	0	1	1	0
3	0	1	0	1
4	1	0	0	1

Child 1: C1

	1	2	3	4
1	1	0	1	1
2	1	0	0	1
3	1	1	1	0
4	0	1	1	0

Child 2: C2

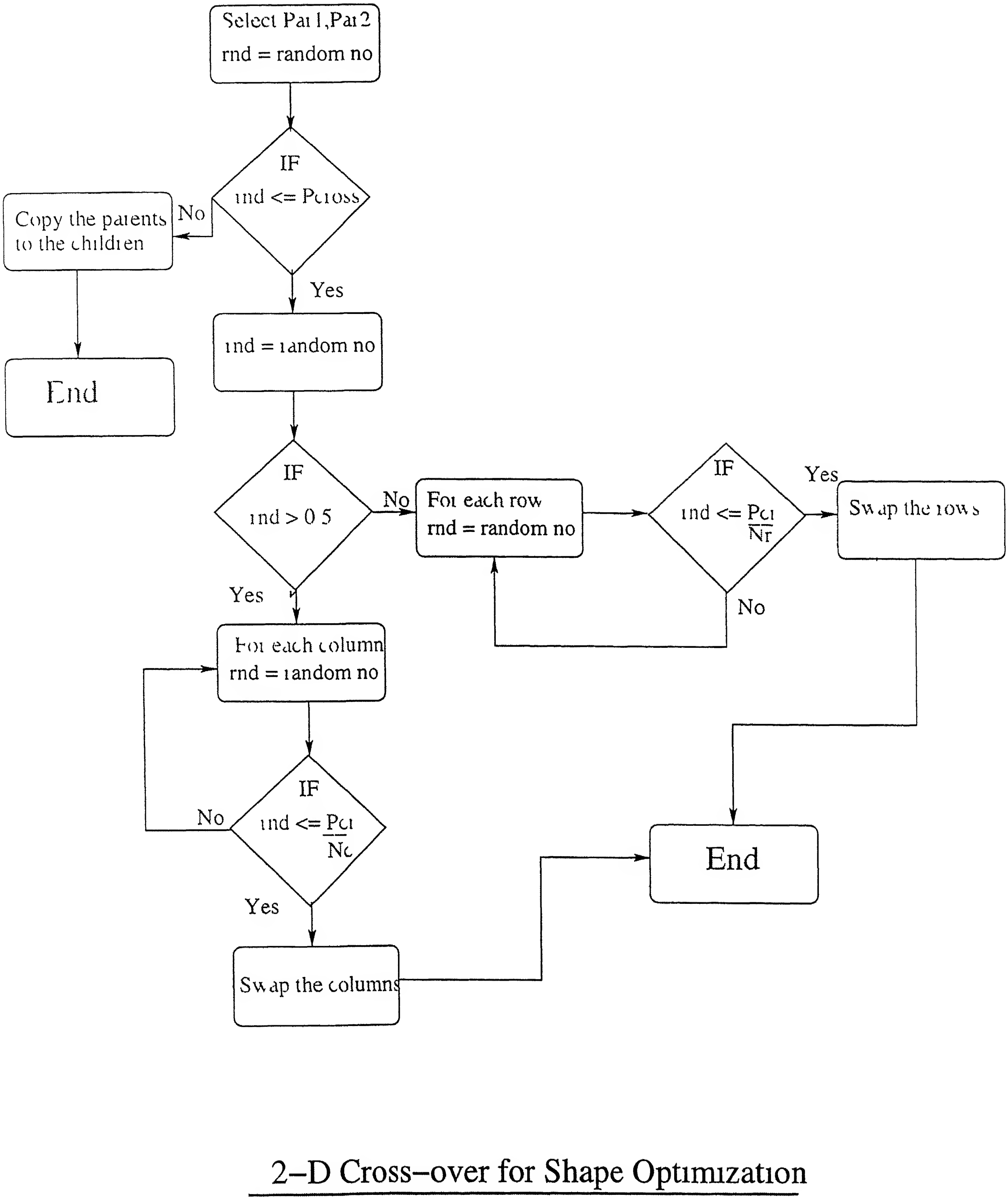

Figure 2 11 Two dimensional crossover

2.9 Closure

In this chapter the methodology of applying GA to the single objective shape optimization problems is discussed in detail The binary strings which can be processed by the GA are converted into meaningful shapes by using the connectivity information and smoothing techniques The constraints are evaluated by finding out the stress and strain values, by using the finite element analysis The constant strain triangle is used as the basic element for the finite element modelling An innovative two-dimensional crossover operator is used for the crossover of the strings This crossover operator respects the geometry and is found very useful here To handle the constraints the selection operator is modified to handle the constraints The hill climbing local search is used to ensure the global convergence of the solution obtained by the GA For local search every time one bit mutation is carried out in a deterministic manner and each time the change is accepted only if it improves the solution The combined method of GA and the local search is termed as the hybrid approach

Chapter 3

Single objective problems and results

In the previous chapter, the methodology of applying GA's to the shape design problems is discussed in detail and the conversion procedure of binary strings into meaningful shapes and vice-versa is established These shapes undergo the finite element analysis for evaluation of stresses and strains In this chapter, the efficacy of the hybrid approach is demonstrated with the help of a number of single objective optimization problems First, all the different problems taken into consideration are described Then the utility of the hybrid approach over the GA alone and the hill climbing alone will be shown with the help of the examples The results obtained by the use of hybrid approach on different problems are presented and discussed The problems taken under consideration are following

- Design of cross-section
- Design of cantilever plate with a point load
- Design of simple supported plate for a number of different loading conditions
- Design of hoister
- Design of bicycle frame for static loading

All these problems are considered for two cases 1) When the weight of the design is not considered for calculation of the stresses and 2) when the weight of the design is also taken into account while finding stresses and strains

3.1 Test problems

In this section different test problems, used to show the efficacy of the proposed approach, are described For all the test problems, a rectangular plate is taken as the basis shape for the design For the test problems following properties of the material are used

Plate thickness	50 mm

Density of material	7800 Kg/m^3
Yield strength	150 MPa
Young's modulus	200 GPa
Poisson's ratio	0 25
Max allowed Displacement	2 mm

For all the test problems other than the problem of design of cross-section following GA parameters are selected

Population Size	30
Crossover Probability	0 95
Mutation Probability	1/String length
No of Generations	100

To determine the effect of weight of the design, weight is also considered as internal loading for each problem Since with the given density, the weight of the plate is very small as compared to the external load applied, it cannot produce much stresses Thus the effect of considering the weight will not be clear with the normal density Hence a very large density is used, which makes the weight of the plate or design into consideration, equivalent to the applied loading and therefore giving substantial contribution to the stresses and strains When weight of the structure is also considered, the density is taken as 1000 times the normal density, as quoted above This analysis of considering the weight effect can be useful in the applications where accuracy requirements are enormous or where the design is carried out at micro level A brief description of different test problems is given as following

3.1.1 Design of cross-section

First problem considered is the design of the cross section which has the maximum moment of inertia to weight ratio The moment of inertia (MI) is calculated about its centroidal axis The distance of the centroidal axis from top of the plate Y_c is calculated as

$$Y_c = \frac{\sum(w_i * y_i)}{\sum w_i} \tag{3 1}$$

where

w_i	Weight of the i_{th} element of the shape
y_i	Distance of the center of gravity of the i_{th} element from the top of the plate

This is an unconstrained problem The weight of the element can be replaced by the area of the element as thickness of the plate and the density of the material is constant The moment of inertia can be calculated as

$$MI = \sum(w_i * (y_i - Y_c)^2) \quad (3\ 2)$$

Desired ratio is given as

$$Ratio = MI / Weight$$

The plate is divided in the grid of 10×10 i e a binary string of length 100 bits is representing the shapes The length and the breadth of the plate is taken as 10 units each In this problem the finite element analysis is not required as no force is applied and stress and strain values are not required *The problem is of maximization of the MI to weight ratio for one axis*

3 1 2 Design of the cantilever plate

For a number of engineering problems, structures of cantilever shape are useful , for example the hanging part of the roofs, etc The next problem taken into consideration is the design of a cantilever plate carrying an end load P = 10 kN as shown in Figure 3 1 The rectangular plate of size 60×100 mm^2 is

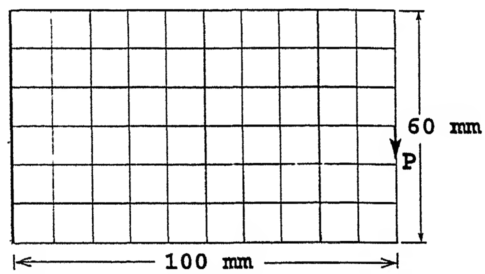

Figure 3 1 The loading and support of the cantilever plate are shown

divided into a grid of 60 small regular elements i e a binary string of length 60 is used to represent the shape The feasible shape satisfies all the constraints The optimal shape is designed for the minimum weight When the weight of the cantilever plate is also considered, the density of the material is taken such that the load due to the weight is comparable to that of externally applied load This requires a very high density which may not be existing but it is used only to find the effect of internal loading

3 1 3 Design of the simple supported plate

The simple supported structures are also one of the most common structures in engineering field Few examples are the bases of the machines, railway tracks, bridges etc The rectangular plate of size 60×100

mm^2 is used for the design. This is divided in a grid of 60 elements. The simply supported plate is designed for a number of loading conditions as given below

Case 1 A point load of P = 10 kN is applied on the middle point of top side of the plate as shown in the Figure 3 2

Case 2 A distributed load of P = 10 kN is applied on the topside of the plate as shown in the Figure 3 3

Case 3 A point load P = 10 kN is applied on the middle point of the lower side of the plate as shown in the Figure 3 4

Case 4 A distributed load P = 10 kN is applied on the lower side of the plate as shown in the Figure 3 5

For all these different loading conditions the feasible structure satisfies all the constraints. The problems are the minimization of the weight

3 1.4 Design of the hoister

For a number of applications like crane hanger etc, hoister type shapes are used for transport of material from one place to another. This structure has a lot of importance as by analysis of this problem certain facts about the sensitivity of the shape towards increase in load can be studied. This also tests applicability of hybrid approach to find shape of difficult and complicated problems of solid mechanics. Here some information about the problem like cavity information is required so that the search is guided to find the solution having a cavity

The rectangular plate of size 60×80 mm^2 is used with 48 elements. Thus the 48 bit long binary string is used to represent the hoister like shape. The loading and support conditions are as shown in the Figure 3 6. Two center elements are forced to made absent (that is, the corresponding bits in the string are always set to zero). The vertical load of 5 kN is applied in the middle of the cavity and horizontal loading of 2 5 kN is distributed over the length of the element. For the single objective problems the objective is the minimization of the weight

3 1 5 Design of the bicycle frame

This is a very interesting problem, which has got a practical importance, as this clearly shows the usefulness of this work for the application purposes. This also helps the designer to conceptualize the new ideas about the shapes. It will be shown that this approach is helpful in finding new creative and more innovative solutions also. Designer can also make design of his requirements by changing the constraints or problem definition

The loading and supports conditions for the bicycle frame like shape is given in the Figure 3 7. In

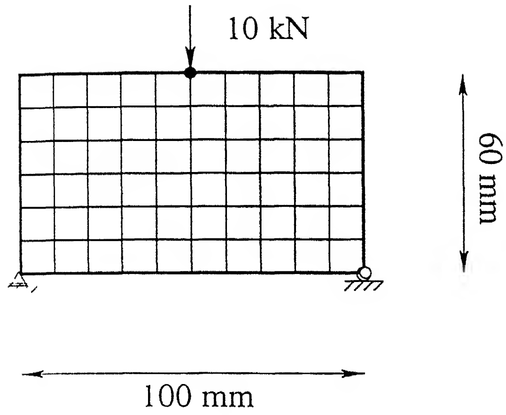

Figure 3 2 Simple supported plate with a point load on top

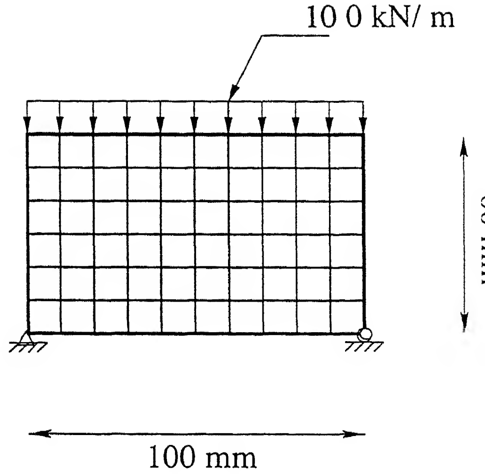

Figure 3 3 Simple supported plate with a distributed load on top

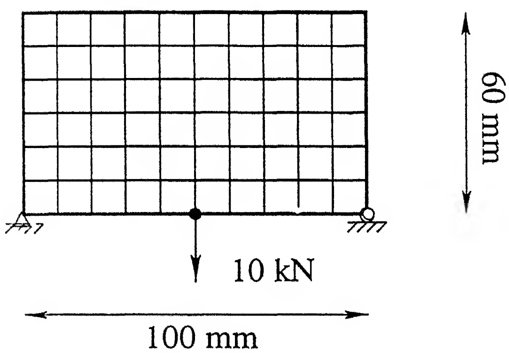

Figure 3 4 Simple supported plate with a point load on bottom

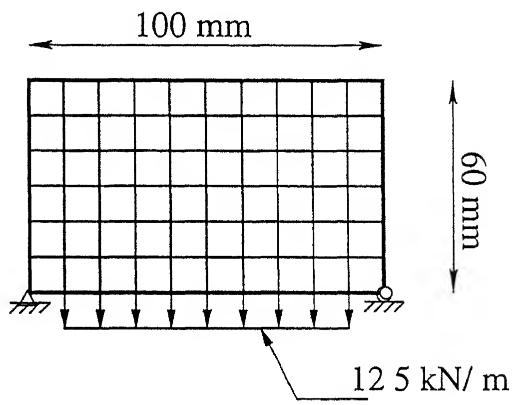

Figure 3 5 Simple supported plate with a distributed load on bottom

this problem the part of the plate is made void to accommodate the front wheel A vertical load is applied on point A where seat is to be placed The frame is supported at two places B and C Point B marks the position of the axle of the rear wheel Another point C serves the purpose of the handle support The filled position is marked as the location of placing the pedal assembly The shape is modelled as a simple supported plate and the design is conceived For this design the material yield stress is 140 MPa, Young's modulus is 80 GPa and Poisson's ratio is 0 25 The maximum allowed displacement is 5mm and the thickness of the plate is taken as 20 mm

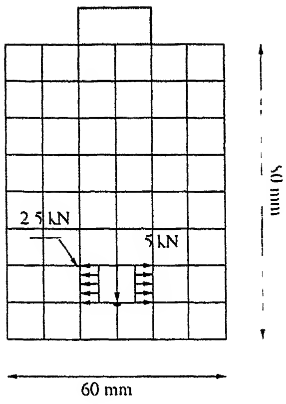

Figure 3 6 The loading and support of the hoister plate is shown

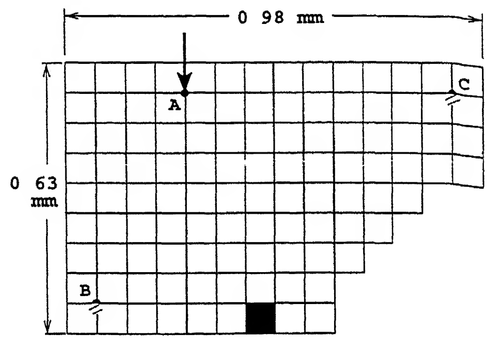

Figure 3 7 The loading and supports for the bicycle frame is shown

3.2 Comparison of the results of random search with hill climbing and hybrid approach

Before using the hybrid approach, it is imperative to show the efficiency of the approach considered for solving the problems and advantage of the suggested approach over other potential methods In this section, this is shown by presenting the results obtained for the test problem of design of cantilever plate, with an objective of minimization of the weight The comparison of the results, obtained by using hill climbing (HC) on a solution obtained by the random search, and the results obtained by using the suggested hybrid approach is presented The results of the random search and GA search are obtained as the intermediate results

To find the solution by GA a population of 30 individuals is taken The GAs are allowed to run for 25 generations Other parameters of the GA are the same as given in the Section 3 1 The best solution found by GA is then recorded and this solution undergoes the single bit hill climbing, a local search meted The solution obtained after the local search is optimized for a single bit i e this solution can not be improved by the random change of any bit

For the hill climbing solution, first the best solution of 750 randomly created individuals is found Here 750 individuals are taken to provide fair opportunity to the hill climbing when compared with GA Since GA simulation runs for 25 generations with a population of 30 so approximately (25 × 30) 750 individuals are processed by GA before giving the best solution That s why the best solution of 750 randomly created individuals is taken for the hill climbing The best solution undergoes the single bit local hill climbing search to reach the final solution

Table 3 1 presents the comparison results for 10 different simulation runs each starting with the random population created by the same random number 10 different simulations are carried out for 10 different equi-spaced random numbers The best solution of each simulation run is recorded and all these good solutions collectively form a set The minimum weight solution of this set of best solutions is the best solution and maximum weight solution of this set is the worst solution of the best solutions The mean median and the standard deviation of this set of best solutions are calculated This weight set is obtained for the random search method, hill climbing on the best solution obtained by the random search genetic algorithm and the suggested hybrid approach All the solutions presented in the table have the same units It is important to mention that the problem is the minimization of the weight

	Random Search	Hill Climbing	Genetic Algorithm	Hybrid Approach
Best Solution	30 00	26 00	24 00	20 75
Worst Solution	36 00	33 25	32 50	25 50
Median Weight	33 13	29 38	27 38	23 75
Mean	32 83	29 45	27 73	23 60
Standard Deviation	2 603	2 681	2 155	1 261

Table 3 1 Comparison of results for 10 different simulation runs

From the solutions obtained for a number of test runs with different random numbers, few important conclusions can be deduced

- The best solution obtained by the hybrid approach in 10 different simulation runs is better than the best solution obtained by other approaches This is also observed that the hybrid approach outperforms other methods in all the cases, starting with the same initial random seed

- The worst solution obtained by the hybrid approach, is better than the worst solution obtained by all other approaches This is even better than the best solution obtained by the random search and hill climbing on this best solution

- The mean of the solutions is also the minimum for the hybrid approach

- The standard deviation for the hybrid search method is lesser than other approaches This shows the lesser variance in the solutions obtained by the hybrid approach and all the solutions tend to

achieve the same optima

- The solutions obtained by GAs is the next best solution after the hybrid approach solution This shows that application of GAs to solve the problem of shape optimization, is a good approach

- The standard deviation of the GA solutions is high This is due to the lack of time provided to GAs to evolve If GAs are allowed to run longer the solutions come close to optima and then the standard deviation will also reduce

It is clear from the results presented in this section that GA can be a good approach to solve the shape design problem On top of GA use of local search helps the solution to reach the global optima quickly and the quality of the solution is also improved This gives an idea of using the hybrid approach for other shape design problems Moreover, the data presented here clearly shows the efficiency of the hybrid approach The mean value, the standard deviation and the best values for hybrid approach are better than any other method Thus this approach can be said as the best approach of all the considered methods for evolving the shapes

3.3 Results for different test problems

The test problems are described in an earlier section In this section, the results obtained from the simulation runs are presented The results obtained by the hybrid approach are shown in all the problems and in some problems where the results obtained by GA are interesting or the results have significant difference, intermediate results are also presented This local search on the GA solution either improves the solution or at least the solution remains as it is But in no case the solution obtained by GA is degraded by this approach This feature makes the hybrid approach more attractive as this approach preserves the best solution

3 3 1 Design of cross-section

For this problem the population size taken is 100 GA simulations are run for 100 generations with a crossover probability of 0 95 and mutation probability of 0 01 before switching to the local search method The shape obtained at the end of the GA runs is shown in Figure 3 8 The local search on this solution further improves the solution The final shape obtained by the hybrid approach is given in Figure 3 9 The moment of inertia to weight ratio of the best solution obtained by the GA is 14 96 units This ratio improves to 15 72 units after the local search on this solution There is a significant improvement in the solution, as the shape is smoothed and the shape is similar to I-section By the knowledge of theory, it is known that I-section has the highest section modulus, and the same is found by the proposed approach Thus, the result is verified, as this follows the theory Since, in this case the

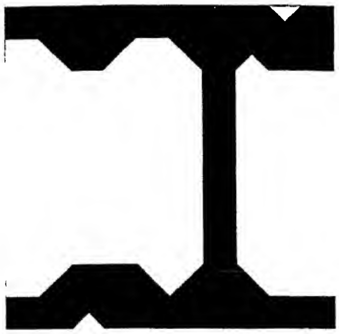

Figure 3 8 GA solution for design of cross-section

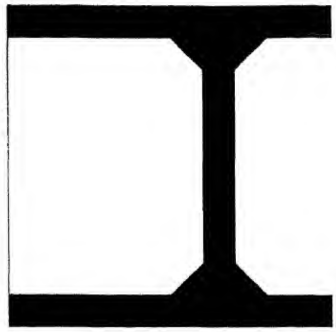

Figure 3 9 Final solution after the local search

formulation is such that there is no special advantage of placing of the vertical component exactly in the middle the proposed algorithm puts it somewhere along the length of the plate That s the reason the solution does not have the vertical bar exactly in the middle

3 3 2 Design of the cantilever plate without considering its weight.

The next problem is the design of the cantilever plate The loading and the support conditions are the same as shown in Figure 3 1 The solution obtained after the GA runs is shown in Figure 3 10 After using the local search, the solution improves and it is shown in Figure 3 11 Table 3 2 compares these two results

Parameter	GA Solution	Hybrid Approach
Weight (units)	22 75	20 25
σ_{max} (Mpa)	136 20	134 68
δ_{max} (mm)	0 147	0 149

Table 3 2 Parameters of solutions obtained by GA and hybrid method

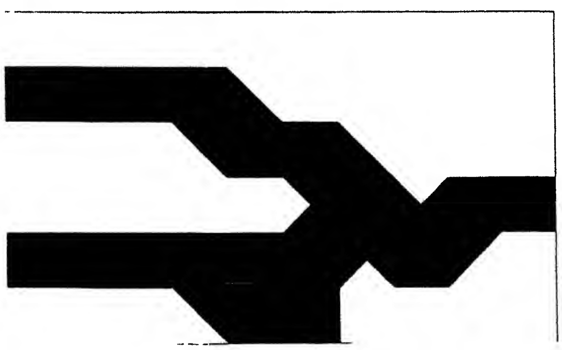

Figure 3 10 GA solution for cantilever plate design

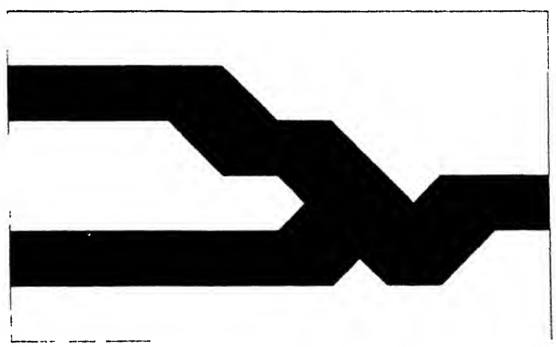

Figure 3 11 Final solution after the local search

The solution obtained by the GA has a small chunk of the material appearing in the last row of the shape This is eliminated by the hybrid approach The solution obtained by hybrid approach, is very similar to the parabolic shape, which is the solution with uniform stress Interestingly, this approach eliminates the two rows from the solution, one from the upper side and one from the lower side, showing the potential of finding the optimal use of the material This is clear that the hybrid approach is able to improve the solution More interesting observations are the values of the maximum stress and strains given in Table 3 2 The solution obtained after using the hybrid approach has lesser maximum stress than the GA solution, but there is slight increase in the deflection This is due to the re-orientation of the shape The results show that the solution obtained is near to the stress constraint This mean that the stress constraint is more important constraint for this problem

3 3 3 Design of the cantilever plate when the weight of the design is also considered

The external loading conditions remain the same as previous problem Only difference is that the load of the structure itself is also accounted as a distributed load, for the calculation of the load The solution obtained after the GA runs is shown in Figure 3 12 After using the local search, the solution improves and it is shown in Figure 3 13

Parameter	GA Solution	Hybrid Approach
Weight (units)	27 50	24 25
σ_{max} (Mpa)	135 26	142 22
δ_{max} (mm)	0 160	0 172

Table 3 3 Parameters of solutions obtained by GA and hybrid method

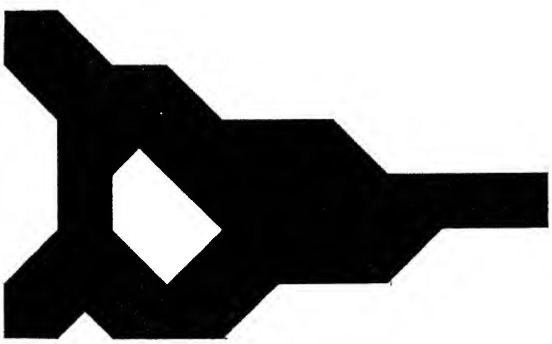

Figure 3 12 GA solution for cantilever plate design (when weight is also considered)

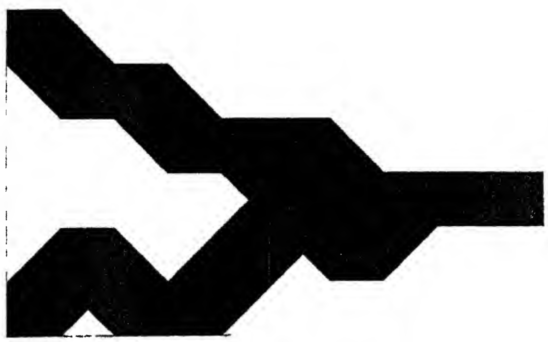

Figure 3 13 Final solution after the local search for the cantilever plate(weight is considered)

Here the shape obtained by GA, has a crossed rib This rib can reduce the stresses But more interestingly this rib is eliminated by the local search in order to save material This causes a slight increase in the stress but the solution remains feasible Though the solution obtained by the hybrid

approach is very similar to the parabolic shape, which is the solution with uniform stress, yet the solution is significantly different from that when the weight of the plate is not considered The solution obtained by the hybrid approach make better use of material as shown in Table 3 3

3 3 4 Design of the simple supported plate when a point load is applied on the top

Next problem taken into consideration is the design of the simple supported plate, when a point load is applied on the topside of the plate in the middle as shown in Figure 3 2

Case 1 Weight of the designed plate is not considered

The solution obtained by the hybrid approach is shown in the Figure 3 14 This solution is similar to the solution obtained after GA runs The numerical data for GA solution and the hybrid approach is given in Table 3 4

Parameter	GA Solution	Hybrid Approach
Weight (units)	20 25	19 50
σ_{max} (Mpa)	49 43	50 57
δ_{max} (mm)	0 021	0 022

Table 3 4 Parameters of solutions for simple supported plate with point load on the topside (Case 1)

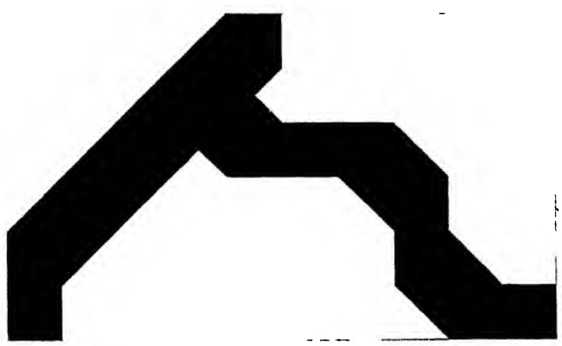

Figure 3 14 Solution obtained after the hybrid approach for the simple supported plate when a point load is applied on topside

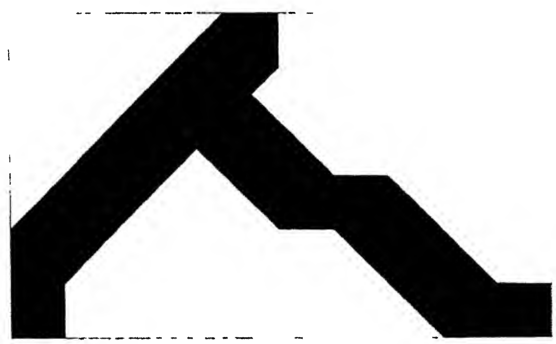

Figure 3 15 Solution after the hybrid approach for the simple supported plate with a point load at the top (weight is considered)

The difference in the weight of the solution obtained by the GA is not much reduced by the solution obtained by the local search Interestingly, the solution obtained by the hybrid approach eliminates all the material other than the arms joining the loaded point to supports The stress and strain values are very small This implies that the thickness of the plate can be reduced which will further reduce the weight and make more effective use of the material It has been observed that for a very small range of fitness there are many solutions which have very little difference in the fitnesses but the solution looks

different from each other This is due to the reason that the search space is discreet, with continuous search space this problem is less likely to happen The shape obtained somehow depends on the initial random population Eventually, if the simulations are allowed to run longer, it may happen that due to genetic drift the solutions may go to one solution But this may require very-very long time before this phenomena is observed

Case 2 Design of the plate when the weight of the design is also considered

The plate considered in the previous section is redesigned for the case when the weight of the plate also contributes to the loading in addition to the externally applied load The shape obtained by the hybrid approach is shown in Figure 3 15

Parameter	GA Solution	Hybrid Approach
Weight (units)	20 00	19 50
σ_{max} (Mpa)	89 44	87 42
δ_{max} (mm)	0 048	0 048

Table 3 5 Parameters of solutions for the simple supported plate when a point load is applied on the topside and weight is also considered

This solution obtained by the hybrid approach is slightly different in weight and the configuration from the solution, when the weight is not considered The solution has same weight and more stresses than the case when the weight of the solution is not considered, as the loading is increased now The weight of the solution can be reduced by using the thinner plate for the design The result obtained by GA is very similar to that by hybrid approach as shown by the numerical values from the Table 3 5

3.3.5 Design of the simple supported plate when a distributed load is applied on the top

In this problem, the distributed load is applied on the top of the plate The loading and the support conditions are shown in Figure 3 3 To make the solution geometrically feasible, all the top row elements must be present

Case 1· The weight of the plate is not considered

The solution obtained by the hybrid approach for the case when the load of the plate is not considered is shown in the Figure 3 16

The numerical data presented in the Table 3 6 shows that the solution obtained by GA and by the hybrid approach is same The interesting thing about the solution is that the left arm is straight joining

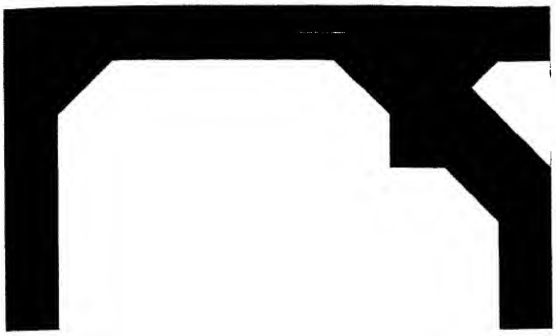

Figure 3 16 Solution after the hybrid approach when the distributed load is applied on the top of simple supported plate

Figure 3 17 Solution for the design of the simple supported plate with distributed load with its weight also considered

Parameter	GA Solution	Hybrid Approach
Weight (units)	23 75	23 75
σ_{max} (Mpa)	32 91	32 91
δ_{max} (mm)	0 016	0 016

Table 3 6 Parameters of solutions for simple supported plate with distributed loading when weight is not considered

the top row with the support node The right arm is slanted This is also clear from the numerical data of the design that the thickness can be reduced as the stress and strains for this design are too less than the allowed value The reduction in the thickness helps in the optimal use of the material The loading can also be increased to make optimal use of the material

Moreover, the design does not seem to be optimal as the loading is symmetric, hence the solution obtained must be symmetric This can happen due to the insufficient time available for the evolution through GA for this problem The solution undergoes local search with one bit change at one time This may not be sufficient to get the global optimal solution in this particular case If the solution obtained by the GA is tried to be optimized with 2-bit flip or more bit flip simultaneously, better solution can be obtained, but the computational effort required is more

Case2. When weight of the designed plate is also considered

The loading conditions now include the weight of the design, as the distributed load The final solution obtained is shown in the Figure 3 17

The numerical data of the solution as presented in the Table 3 5 This clearly shows there is no improvement in the solution obtained from the GA solution Here the solution is having its right arm straight And the left arm is somewhat slanted The slant angle is more than the case when the weight of the solution is not considered The same results are obtained for the many simulation runs with

Parameter	GA Solution	Hybrid Approach
Weight (units)	24 25	24 25
σ_{max} (Mpa)	106 48	106 48
δ_{max} (mm)	0 087	0 087

Table 3 7 Parameters of solutions obtained for the design of simple supported plate with a distributed load applied on the topside of plate when weight is also considered

different random numbers The solution requires more GA runs to reach the global optima

3 3 6 Design of the simple supported plate when a point load is applied on the bottom-side

The simple supported plate is designed for the case when a point load is applied on the bottom side as shown in the Figure 3 4

Case 1 When weight of the plate is not considered

Figure 3 18 Solution after the hybrid approach for the simple supported plate with a point load on the bottom side

Figure 3 19 Solution after the hybrid approach for the point load on bottom side when weight is considered

The solution obtained by the application of the hybrid approach is shown in the Figure 3 18 The numerical data for the problem is given in the Table 3 8

Parameter	GA Solution	Hybrid Approach
Weight (units)	10 00	10 00
σ_{max} (Mpa)	136 67	136 67
δ_{max} (mm)	0 081	0 081

Table 3 8 Parameters of solutions obtained for the design of simple supported plate when a point load is applied on the bottom side of the plate

The results shown in the Table 3 8 clearly show that the GA finds the global optimal solution This solution is the global optimal solution as the shape cannot remain feasible with lesser number of elements

This solution has interestingly eliminated all the four rows from the top without having any problem information To connect the node where the load is applied to the support nodes, the path taken is the shortest path So the solution becomes the global best solution Thus the final solution obtained through the hybrid approach is the same as the GA solution

Case 2 When the weight of the designed plate is also considered

The same plate is redesigned for the case when the weight of the designed plate is also considered The result obtained by the hybrid approach is shown in the Figure 3 19 The numerical data for the problem is presented in the Table 3 9

Parameter	GA Solution	Hybrid Approach
Weight (units)	13 75	13 75
σ_{max} (Mpa)	114 56	114 56
δ_{max} (mm)	0 054	0 054

Table 3 9 Parameters of solutions obtained for the loading on the bottom side when the weight of simple supported designed plate is also considered

Here the shape is considerably different from the result in the previous case As the loading is increased due to the weight of the design, the previous shape does not remain sufficient to sustain the load So the shape is changed More interestingly, the solution has got two cavities Here also the result does not show any improvement by the local search It means GA is able to find the optimal solution in this case The improvement is only possible by reduction in the thickness Table 3 9 gives the value of stresses and strains for the optimal solution obtained by GA and by hybrid approach

3 3 7 Design of the simple supported plate when a distributed load is applied on the bottom

The loading and the support conditions are shown in the Figure 3 5 The problem requires all the elements of the last row to be present in the solution to make it geometrically feasible

Case 1 When weight of the plate itself is not considered

The design is carried under the action of external loads only The solution obtained by the hybrid approach for this problem is shown in the Figure 3 20

The numerical data for the solution is given in the Table 3 10

The shape obtained by the hybrid approach eliminates the top rows to reduce the weight Since all the elements in the bottom row must be present, to make the geometry feasible and there is no extra

Figure 3.20 Solution by the hybrid approach when the distributed load is applied on the bottom side of simple supported plate

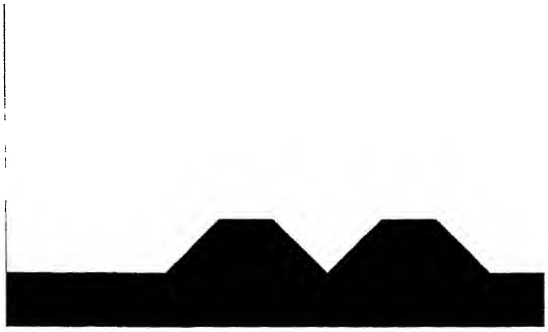

Figure 3.21 Solution after the hybrid approach for the simple supported plate with the distributed load on bottom side when weight is considered

Parameter	GA Solution	Hybrid Approach
Weight (units)	10.00	10.00
σ_{max} (Mpa)	78.20	78.20
δ_{max} (mm)	0.052	0.052

Table 3.10 Parameters of solutions obtained for the design of simple supported plate with a distribued loading applied on the bottom side when the weight is not considered

element in the solution, this solution can be said a global optimal solution. The data shows that GA is able to find the global optimal solution. Since no additional element is present there is nothing to improve by the local search, hence the same solution is retained by the local search. The weight can only be reduced by reducing the thickness.

Case 2 Weight of the designed plate is also considered

The solution for this case is shown in the Figure 3.21. The solution is very interesting as this shows the trend of placing the material for this problem when the loading is increased. The numerical data for the solution show that the global optimal solution for this has more weight than the solution obtained for the case when weight of the designed plate is not considered.

Parameter	GA Solution	Hybrid Approach
Weight (units)	14.00	14.00
σ_{max} (Mpa)	142.69	142.69
δ_{max} (mm)	0.069	0.069

Table 3.11 Parameters of solutions obtained for the design of simple supported plate with a distributed loading applied on the bottom side when the weight is also considered

Since increased loading is present, hence the solution requires more number of elements to sustain the load. The result obtained by GA does not improve by the local search.

3 3 8 Design of the hoister

The loading conditions and the supports are shown in the Figure 3 6 This problem is a very interesting problem as the expected solution have a cavity The simulation results after completing the process of GA and the local search are given in as following

Case 1 Weight of the hoister itself is not considered

Here the weight of the designed hoister is not considered The solution obtained is shown in the Figure 3 22

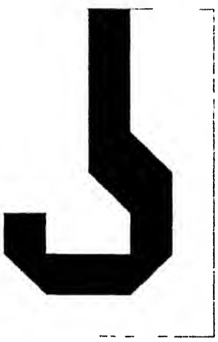

Figure 3 22 Solution after the hybrid approach for the design of hoister

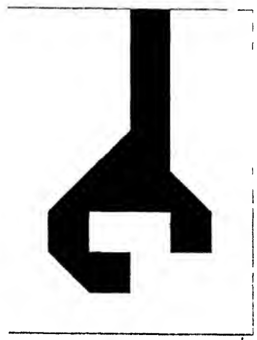

Figure 3 23 Solution after the hybrid approach for the design of the hoister when weight is considered

The solution is very interesting in the nature, as a perfect hook is obtained The solution is not a closed loop solution, which has lesser stresses but the weight is more The result eliminates the leftmost and rightmost column and the bottom most row This hoister can be used for carrying the goods as in the cranes etc This solution is the least weight solution Most interesting observation for this problem is that for the same fitness, three different solutions were obtained This means the configuration of the solutions are dependent on the initial random population The numerical data for the design considered is given in the Table 3 12

Parameter	GA Solution	Hybrid Approach
Weight (units)	12 50	11 00
σ_{max} (Mpa)	67 74	68 77
δ_{max} (mm)	0 072	0 081

Table 3 12 Parameters of solutions obtained for the design of the hoister

The result in the Table 3 12 shows that the solution obtained after local search is improved

Case 2 When weight of hoister is also considered

Now, the same hoister is designed when the load of the designed hoister is also considered as having some effect on the loading The result obtained is shown in the Figure 3 23 Now the result is not like a hook Instead a small difference in the configuration is observed

Parameter	GA Solution	Hybrid Approach
Weight (units)	15 75	11 00
σ_{max} (Mpa)	76 43	88 82
δ_{max} (mm)	0 048	0 087

Table 3 13 Parameters of solutions obtained for the design of hoister when the weight is also considered

The Table 3 13 shows the numerical data obtained for the design problem The comparison of the GA result and the final solution data shows that the solution obtained after the local search is not only improved in the weight , but also it rearranges the material such as the maximum stress and the maximum deflection are also reduced Most interesting thing about this solution is that this solution also has got the weight same as the solution in the case when the weight of the designed plate is not considered But the configuration has now changed This geometry has got the advantage of having lesser moment as compared to the previous case solution This shape has got both the arms carrying the horizontal load are connected to the body so these tend to neutralize the effect of each other making the shape more stable This change in configuration helps in reducing the stresses when the weight of the design is also accounted for the loading

3 3 9 Design of the bicycle frame

This is the most interesting problem as this is closest to the real world problem The solution of this problem shows the ability of the suggested approach in solving the real world problems The support and the loading conditions are shown in the Figure 3 7

Case 1 Weight of the frame is not considered

First, the design exercise is carried out when the weight of the design does not account for the loading The best solution obtained by the hybrid approach is shown in the Figure 3 24

Parameter	GA Solution	Hybrid Approach
Weight (units)	35 25	28 50
σ_{max} (Mpa)	38 03	32 13
δ_{max} (mm)	0 157	0 118

Table 3 14 Parameters of solutions obtained for the bicycle frame design problem

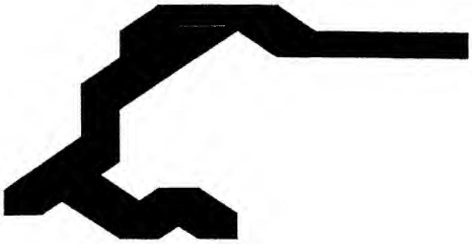

Figure 3 24 Solution after the hybrid approach for the design of bicycle frame

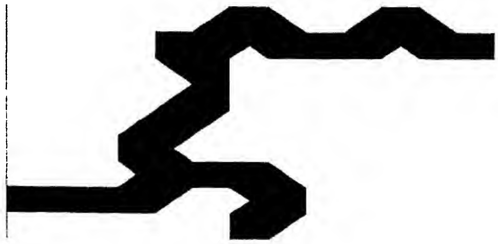

Figure 3 25 Solution after the hybrid approach for the design of the bicycle frame when its weight is considered

The best result is very much different from the conventional shape of the frames This has got a structure which has eliminated the arm connecting the paddle support block and the handle support nodes The shape of the bicycle with this frame is shown in the Figure 3 26 This solution can be a potential candidate of design in the search of the new models of the bicycle But this designed is required to be checked for dynamic loading conditions The numerical data from the Table 3 14 shows that the stresses of the solution are very small Thus the material can be taken from the lateral direction without affecting the shape The data also shows the reduction in the weight of the solution obtained by the GA by the local search

Case 2 When weight of the frame is also considered

Here the design is nearer to the practical conditions The weight of the solution is also considered, but the density of the material used is increased by 200 times than that taken in the case when weight of the design is not considered This is done to have the internal load due to the weight of the same order of the external load

Parameter	GA Solution	Hybrid Approach
Weight (units)	42 00	31 25
σ_{max} (Mpa)	86 39	79 53
δ_{max} (mm)	0 709	0 910

Table 3 15 Parameters of solutions obtained for design of bicycle frame when weight is also taken into consideration

The solution obtained by the hybrid approach is shown in the Figure 3 25 The solution has more weight than the previous case Table 3 15 presents the numerical data for the design The stress and strain values are more for this case than the previous case but still the solution can be improved by the reduction in the thickness It is very clear from the numerical data obtained that the local search

eliminates some part of the material which does not participate in the stress sharing In the process the deflection of the design has increased

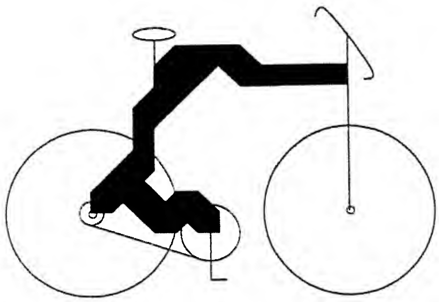

Figure 3 26 Bicycle shape for the frame when the weight is not considered

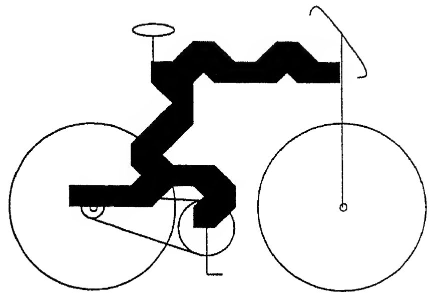

Figure 3 27 Bicycle shape for the frame when the weight is considered

3 4 Closure

This chapter deals with the description of the various shape design problems solved to show the efficacy of the approach Then the results obtained by applying the hybrid approach to solve the test problems are presented and discussed The comparison of the local search method with the hybrid method is used to show the worth of the proposed approach The results support the concept of using the hybrid approach to solve the shape design problems The results obtained for the single objective optimization reveal some interesting facts The stress constraint is found to be the active constraint in all the problems The solutions obtained by the considered approach do not have any control on the configuration It is possible to obtain different shapes which possess the same fitness value Thus the results provide sufficient motivation to solve the problems as the multiple objective problem Then the variation in the weight as well as the effect of configuration on the design can be studied more easily The trend of weight change and optimal method of placing the weight in the design, such as it is used in the most efficient way, can also be obtained by the study of the problems for the multi-objective cases

Chapter 4

Multi objective GA for shape design problems

In the second chapter, it is discussed how GA's can be applied for a single objective problem. In this chapter it will be shown how the Pareto-optimal solutions can be obtained for the shape design problems with more than one objective functions. This study deals with the case when there are *two* objectives of conflicting nature. The problem differs from the single objective problem in the sense that the number of objectives is more than one, and these are of conflicting nature. Then no single optimal solution can be said the best solution, instead a number of solutions which lie on *Pareto-optimal Front* are found.

For the classical methods there is a single way of solving these types of problems— convert the multi-objective problem into a single objective problem. There are a number of ways of doing this: weighted sum approach, ϵ-perturbation, min-max method, goal programming and others. But none of these methods are reliable as the solution largely depends on the parameter settings to formulate the single objective problem. Besides this the classical methods give one solution at a time, so to get a number of solutions, many simulation runs are required. Multi-objective GA's do not face these problems while dealing with a number of objectives as well as a population of individuals. A number of multi-objective optimization techniques [6, 15, 21, 24, 29, 32, 38] are suggested and it has been shown that these evolutionary studies has the potential to achieve the Pareto-optimal front as well as they can reach the global Pareto-optimal front [37].

In this chapter a hybrid method based on the combination of evolutionary algorithms and the local search method is suggested to find the optimal shape design problems. The number of solutions obtained after the evolutionary search is more than required, so these are reduced to a practically suitable number by using a clustering method.

4.1 Multi-objective optimization and pareto optimality

In a multi-objective optimization there are more than one objectives If these objectives have the optimal solutions distant to each other the objectives are said to be *conflicting* in nature Multi-criterion optimization with such conflicting objectives results in a set of optimal solutions Many solutions are optimal because no solution can be considered better than other with respect to all the objectives This solutions set is better known as Pareto- optimal solutions

Definition 2 *Pareto-optimal Front is the set of solutions which are equally good as compared to any other solution of the set i e any solution on the front can not be selected when compared with other without being biased towards other Mathematically it can be given as*

A decision vector $x \in X_f$ is said to be non-dominated regarded to a set $A \subseteq X_f$ if

$$\nexists a \in A \quad a \succ x$$

If it is clear within the context which set A is meant, it is simply left out Moreover, $\mathbf{x}$ *is said to be Pareto-optimal if* $\mathbf{x}$ *is non dominated regarding X_f* "

Figure 4 1 gives a better illustration of the concept of Pareto- optimality through a example problem of trade-off between two conflicting objectives namely, cost and the accident rate, both to be minimized The point A represents the solution with a very small cost but highly accident prone Another solution B represents the solution very safe but very costly Here both the solutions are good with respect to the one objective, but worse in other objective There exist a lot of solutions (like solution D) which also belong to the Pareto-optimal set and have equal importance to a unbiased decision maker All these solutions (lying on the thick solid line) are known as the members of *Pareto-optimal Front* and this solid line is called *Pareto-optimal Front*

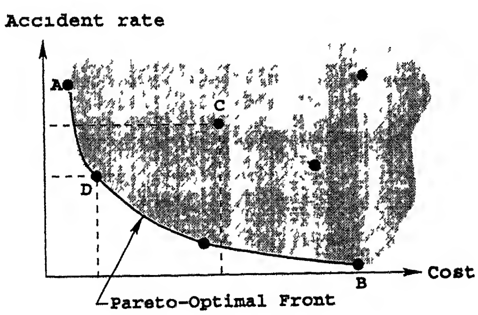

Figure 4 1 The concept of pareto-optimal solutions is illustrated

4.2 Dominance

This is a very important concept so far as the discussion on multi-objective problems is concerned. This forms the back bone of the ranking strategy. For constrained and unconstrained dominance definition varies. Both of the definitions are given as following

4.2.1 Dominance for unconstrained problems

Definition 3 *If there are N objective functions and two individuals, "a" and "b" are compared then*

$$
\begin{array}{l}
a \succ b \ \texttt{(a dominates b)} \qquad \textit{if}\ f(a) > f(b) \\
\qquad f(a) > f(b)\ \texttt{means that} \qquad \forall i \in N \qquad f_i(a) > f_i(b) \\
\textit{or} \\
a \succeq b\ \texttt{(a weakly dominates b) if} f(a) \geq f(b) \\
\qquad f(a) \geq f(b)\ \textit{means that} \\
\qquad \forall i \in N \qquad f_i(a) \geq f_i(b) \\
\qquad \textit{and} \qquad \wedge\ i \in N \qquad f_i(a) > f_i(b) \\
\textit{or} \\
a \sim b\ \textit{(a is indifferent to b)} \qquad \textit{if}\ f(a) \not\geq f(b) \\
\qquad \wedge \qquad f(b) \not\geq f(a)
\end{array}
$$

In simple words it can be said, that "a dominates b" if and only if for all function values, the function value of a is either better or at least equal to that of function value of b, and at least one function value of a is better than b. If all function values of a are equal to those of b or if in some function values a is better and for some function values b is better, a and b are said to be *indifferent* to each other.
This can be understood with the help of the Figure 4.1. It is observed that the solutions which are not on the Pareto-optimal front (like solution C) also exists. If it is compared with solution A, according to definition 3 both are indifferent. But if it is compared with another solution D, it is found to be dominated by solution D. Since for a solution C, there exists a solution D in the search space, which dominates it, this solution cann't be in Pareto-optimal Front.

4.2.2 Dominance for constrained problems

In the multi-objective cases where the constraints are present, definition of "dominance" is redefined to accommodate the constraints.

Definition 4 *The definition of dominance when more then one constraints are present is given according to the following conditions*

Condition 1: *If individual* A *is feasible and individual* B *is not then A dominates B*

Condition 2 *If individual* B *is feasible and individual* A *is not then B dominates A*

Condition 3: *If both of the individuals are infeasible then the individual with lesser violation of constraint(s) dominates the other*

Condition 4 *If both of the individuals are feasible or equally infeasible, dominance is decided according to the definition 3*

4.3 Representation scheme

For multi-objective problems the representation scheme (Section[2.2]), finding connectivity (Section[2.2.2]), smoothing operation (Section[2.2.3]), material selection (Section[2.3.3]), and finite element analysis (Section[2.7]) are the same as discussed in second chapter. Here the fitness is represented as a vector as multiple objectives are present instead of a single objective. So the selection method is also different. To solve such problems we deviate from simple GA and the specialized algorithms— multi-objective genetic algorithms (MOGA)—which can handle multiple objectives are used. For this purpose a specific MOGA *Non-dominated Sorting Genetic Algorithm -II* (NSGA-II) [6] is used.

4.4 Multi-objective problems

For multi-objective problem of shape design problem, two minimization type objectives are considered. Three constraint need to be satisfied to have a safe design.

Objective functions

- First objective is **weight** of the object. It is defined as

$$Weight = Area * Thickness * Density \tag{4.1}$$

  This is to be **minimized**.

- Second objective is **stiffness**. It is defined as

$$Stiffness = \frac{1}{[\delta_{maximum}]}$$

  This objective is to be maximized. This can also be formulated as minimization of maximum displacement. Then both objectives are *minimization* type.

Both of these objectives are of conflicting nature, as more material means more weight but loosely saying more stiffness and lesser material means lesser weight but low stiffness. So there is no single optimum solution, instead it gives a set of optimum solutions which lie on the Pareto-optimal front.

Constraints

Similar to the single objective case three constraints are there but there is a little difference in the constraint violation conditions

- Geometry Constraint This is same as described in Section 2 3 2
- Stress Constraint This is also same as described in Section 2 3 2
- Displacement Constraint Here we put a upper limit on the allowed displacement If the max displacement is more than the specified value the solution is not accepted But now this constraint does not contribute in finding the error

4 5 Non-dominated sorting genetic algorithm-II

This algorithm has the features required for a good MOGA It is shown elsewhere [7] that it can maintain the diversity on the Pareto-optimal front as well as can converge to global Pareto-optimal Front Besides this the time order complexity of this algorithm is shown as $O(MN^2)$ [7] The steps involved in the algorithm are given here

Step 1 Initialize the population

Step 2 Calculate the fitness

Step 3 Rank the population using the dominance definition defined in Section 4 2 or in Section 4 2 2 as applicable for non-dominated sorting [32]

Step 4 Calculate the crowding distance [Section 4 5 1]

Step 5 Do selection using crowding comparison operator [Section 4 5 2]

Step 6 Do crossover, mutation and generate the intermediate children population

Step 7 Combine the intermediate population and parent population and do non-dominated sorting and find crowding distance

Step 8 New parent population is formed by accepting solutions from the first (best) non-dominated front and continuing to other fronts successively till the new population exceeds the number of individuals as the parent population had

Step 9 The solutions from the last accepted front are sorted according to the crowding distance and as many individuals are selected, which make the population of new parent population same as old one

Step 10 Repeat Step 5

4.5.1 Crowding distance

In order to preserve the diversity along the front, the selection of individuals which are very near to each other must be discouraged. To do the same this concept of crowding distance is used. The crowding distance is measured as the distance of the biggest cuboid containing the two neighbouring solutions of the same Pareto-optimal front in the objective space as depicted in Figure 4.2. The crowding distance

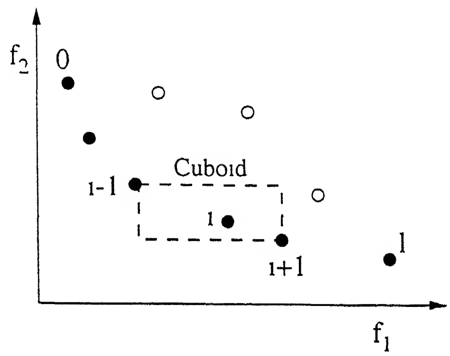

Figure 4.2 The crowding distance calculation is shown

is calculated by first sorting the solution set with respect to each objective function and finding the distance between the immediate neighbours. The solutions on the ends are assigned a large value of crowding distance. This helps in preserving the extremities of the solution set. Then the sum of these distances for each solution is called as crowding distance of that particular solution. For more details about the crowding distance refer to [6]

4.5.2 Crowding comparison operator

Since the diversity among the solutions is important, a *crowded comparison operator* is defined using a relation $\prec_n$ as following

Definition 5 *Solution i is better than solution j in relation $\prec_n$ if $(i_{rank} < j_{rank})$ or $((i_{rank} = j_{rank})$ and $(i_{distance} > j_{distance}))$*

where $i_{distance}$ is the crowding distance of the i_{th} solution and i_{rank} is the rank of the i_{th} individual

4.5.3 Selection

In the single objective problems the selection is based on the basis of the fitness of the individuals but in multi-objective optimization problems there are more than one objective functions, so selection cannot be done on the basis of fitness as both solutions may be better than each other in different objectives. Selection in multi-objective problems is done according to crowding selection operator described in the

previous section. Better rank individuals are given preference over the individual with worse rank of the two. If both individuals have the same rank, the individual with more crowding distance is selected.

4.5.4 Elitism

Elitism is a way of preserving the good solution found so far. This helps the evolution strategy to converge faster to the global Pareto-optimal front. To employ the elitism, old and the population generated after selection, crossover and mutation are combined. This combined population is ranked and this population is halved, selecting the best individuals. This way all the good individuals are preserved.

4.6 Hybrid methods

The evolution strategy selected for solving multi-objective problem is found to perform very well on the test problems, which are of very complex nature. In this section, a hybrid strategy combining the evolutionary method with classical method, is suggested to make the approach more practical by

1. ensuring convergence to the Pareto-optimal front

2. reducing the size of the obtained non-dominated set

4.6.1 Better convergence

In a real world problem the knowledge of global Pareto-optimal front is usually not known. Though NSGA-II has shown the ability to find the global Pareto-optimal front for a number of test problems [7, 8] but it is always helpful to enhance the probability to reach the true global Pareto-optimal front by using the a local search method at the rear end of the algorithm. The local search algorithm requires single objective function, hence a weighted sum of objectives is used to convert the multiple objectives into a single objective. The single objective is defined as

$$F(x) = \sum_{j=1}^{M} \bar{w}_j^{\mathbf{x}} f_j(\mathbf{x}) \tag{4.2}$$

where

$f_j(\mathbf{x})$	j^{th} objective function
$\bar{w}_j^{\mathbf{x}}$	weight corresponding to the j^{th} objective function

In the equation 4.2, if the scaled function values are not used and if the magnitude of the function values are not of same order, the obtained sum may be biased towards one function though it is not desired and intended. To avoid this difficulty the fitness function values are scaled between 0.1 and 1.0

Scaling of the function values is done according to the following formula

$$f_i^{scaled} = 0.1 + \frac{(f_i - f_i^{min})}{(f_i^{max} - f_i^{min})} \tag{4.3}$$

where

f_i^{scaled}	Scaled value of i_{th} objective function
f_i	Value of i_{th} objective function
f_i^{min}	Minimum value of i_{th} objective function over the population
f_i^{max}	Maximum value of i_{th} objective function over the population

The biggest obstacle to this single objective problem is in finding the weight vector as it is obvious from the equation 4.2 that the weights affect the sum of fitness. Any setting of the weight vector will lead to a different optima. There can be a number of ways of finding weight vector like giving equal importance to each function value or let the user specify the weights etc. But all these methods suffer from the very fundamental problem of requirement of the problem information before hand. If the weight is not chosen properly, it may mislead the search and guide to any local optimal point. The proposed method of weight calculation is elaborated in Section 4.6.4. Since this method of weight calculation gives the weights different from the user specified weights so these are called as pseudo-weight vector $\bar{w}$.

Optimization of the weighted sum of the fitnesses, $F(x)$ is done using the hill climbing local search method [Section 4.6.3]. This optimization will produce a Pareto-optimal or a near Pareto-optimal solution as illustrated in Figure 4.4. This is true for the convex Pareto-optimal regions. Since independent local search methods are tried for all different solutions of the Pareto-optimal set, it may happen that all the solutions obtained by local search are not non-dominated to each other. Thus, once again a non-dominated sorting is performed to obtain the non-dominated set.

4.6.2 Reducing the size of non-dominated set

In the multi-objective problems, a number of solutions are obtained, which are represented as different points on the Pareto-optimal front. These gives a number of solutions to the designers or the designer will have a lot of choice, but practically it is not feasible for the designer to take a look on all the possible solutions. So the number of solutions must be reduced to a number on which, a designer can concentrate to decide a final solution according to his/ her requirement. It may be a question here—When so many solutions are not wanted, why to obtain so many by GA, why not reduce the population size? The answer to this is that it can not be done. GA's work with the population. To find the global optimum it requires a optimum level of population [17].

Very small population will not have the potential to search ahead and reach the global optimum or it may prolong the convergence. It will converge very early to a local optima. If very large population

is considered our search will reach the global optimum but it will be consuming a lot of time as this process is computationally very expensive due to finite element analysis of all the shapes and then the non-dominated sorting Besides this again a lot of solutions will be present, which pose the same problem to the designer of having too many solutions

To reduce the number of the solutions, some solutions, which can be improved or which forms a cluster can give way to other solution which are good as well as have diversity in the different objectives This exercise is carried out to some extent by the strategy of local search and then mainly clustering (Section 4 6 5] takes care of it

The complete procedure of the proposed hybrid method is shown in Figure 4 3 Starting from the problem the evolutionary strategy yield the primary solution set This is processed by the local search method which leads to a better converged set of solutions Non-dominated solutions are extracted from this set Clustering reduces the number of the solution to a number required by the designer

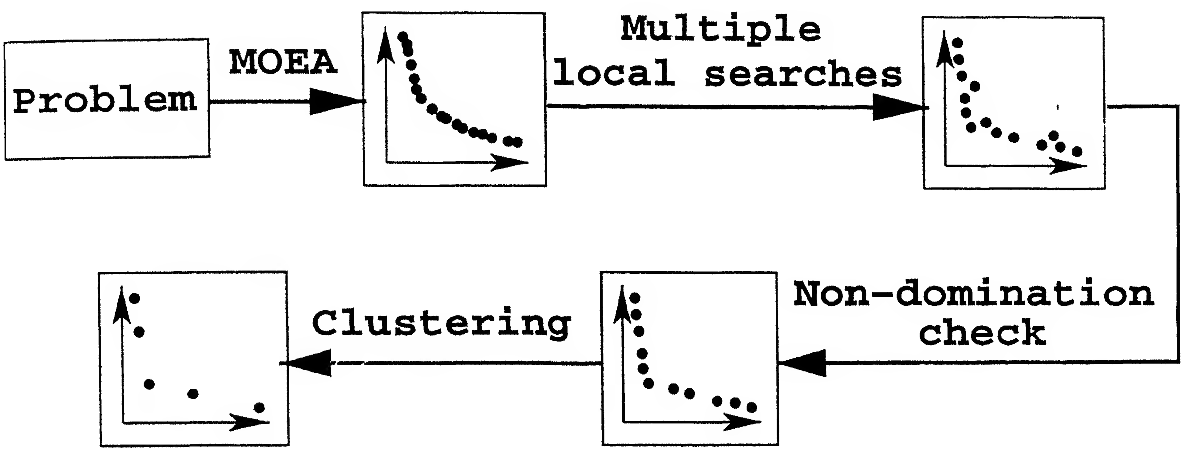

Figure 4 3 The proposed hybrid procedure of using a local search technique, a non-domination check and a clustering technique is illustrated

4 6 3 Hill climbing

The Hill climbing method for the multi-objective process is given here This is a method of finding the optima starting from a initial guess This initial guess is provided by the solution obtained by MOGA runs This is more like a steepest descent search, trying to find the solution in the global basin, when it is expected to be near the global optimum The proposed method is as following

1 Select an individual and calculate the unrefined weighted sum of the scaled fitness of this solution

2 Mutate one bit each time in the individual string

3. Extract the shape from the new string

4. Calculate the stresses, displacement and fitness values for this new solution (shape)

5. Calculate the weighted sum of the scaled fitness

6. Compare the unrefined sum and the new obtained solution. If there is improvement in the new solution, accept the change else reject it

7. In case of rejection restore the previous bit value

8. Repeat the process till all the bits are mutated once

9. Calculate the total number of bit changes. If there are no bit changes, terminate the hill climbing process for this individual, else repeat Step 2, from the first bit of the individual string

10. If all the individuals have undergone this hill climbing process, terminate the hill climbing, else repeat Step 1

The Figure 4.4 illustrates the basic phenomenon of hill climbing. The basic idea behind this exercise is that once the solutions close to the true Pareto-optimal front are obtained from evolution strategy, the local search method with different emphasis of objective functions can be used as an attempt to reach the true Pareto-optimal front.

This is also of the importance from the point of view of reduction in computational effort. It may happen that the search algorithm is near the true Pareto-optimal front but it may require more number of generation to reach the global front, making the use of evolutionary method alone computationally expensive. Here we can stop the process of evolution at this step and let the local search method take it over to find the global Pareto-optimal front speedily.

4.6.4 Weight calculation

Here, a strategy similar to that proposed by Deb, 2000 [10], which eliminates the requirement of weight determination from the user, is used. In fact, this methodology determines the weight by using the fitness values of the solutions, hence eliminating the problem of weight determination by the user. Weights are calculated on the basis of their function values. Two methods of weight calculation are applied here.

- (A) Fixed weight method

- (B) Continuously updated weights method

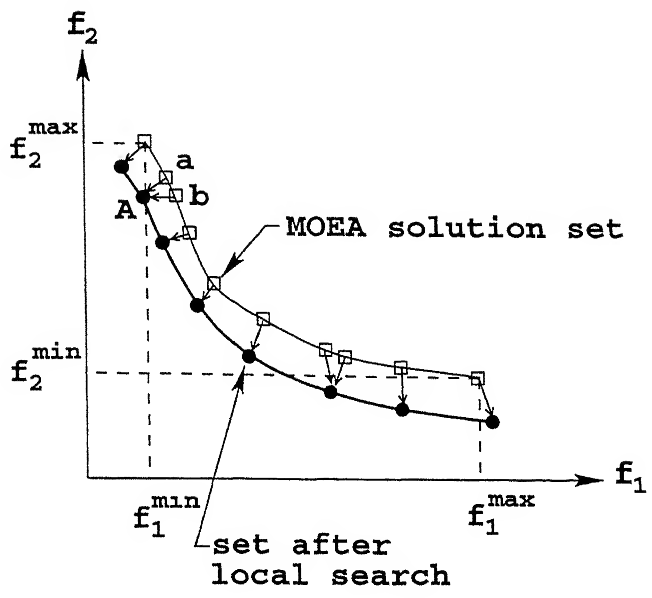

Figure 4.4 Finding better solutions through local search method

A brief description of both the methods is given here

Case A *Fixed weight method*

The weights are calculated by using the fitness values of the individuals and using a linear mapping to find out the weights. First, the extremes of the front on all the functions is sought and the fitness function values are scaled using equation 4.3. Following formula is then used to calculate the weights of the different individuals

$$\bar{w}_j = \frac{(f_j^{\max} - f_j(\mathbf{x}))/(f_j^{\max} - f_j^{\min})}{\sum_{k=1}^{M}(f_k^{\max} - f_k(\mathbf{x}))/(f_k^{\max} - f_k^{\min})} \tag{4.4}$$

where

$\bar{w}_j$	Weight for the j^{th} function
f_j^{max}	Scaled maximum value of the j^{th} function in the population
$f_j(\mathbf{x})$	Scaled value of the j^{th} function
f_j^{min}	Scaled minimum value of the j^{th} function in the population
M	Number of different functions

In this calculation of the weights, the objective functions are of minimization type. When a solution is close to f_j^{min}, the numerator becomes one, causing a large value of the weight for that function. And that function is given a lot of importance. Figure 4.4 shows the embedded concept. If the Pareto-optimal front does not have the bias toward any particular region of the front, this strategy works fine and approximates the weights fairly well. But if the Pareto-optimal front is having a bias towards any region the linear approximation of the weights does not perform well

Case B *Continuously updated method*

For some problem which have the bias towards the some part of Pareto-optimal front, it was observed that the previous weight calculation strategy does not perform well, as almost all the solutions try to converge near to one solution and form a cluster thus taking away the diversity This is undesirable It is due to the reason that the weighting function given in equation 4 4 tries to map the region with a linear function although the Pareto-optimal front may have a bias towards a particular region thus giving equal importance to all the solutions But this method does not give the most optimum weight vector corresponding to the different solutions So in order to reach near to the natural weights of the solution on a Pareto- optimal front a continuous updated method is proposed

This equation 4 4 is based on the assumption that the solutions at the extremities are true extremities But it may be a true extremity under some constraints, not a free extremity This fact is taken care while finding the weights of individuals other than extremities, in the continuous updated weight calculation method This method is more clear with the help of the Figure 4 5 for two objectives For more than two objectives a similar procedure can be adopted easily

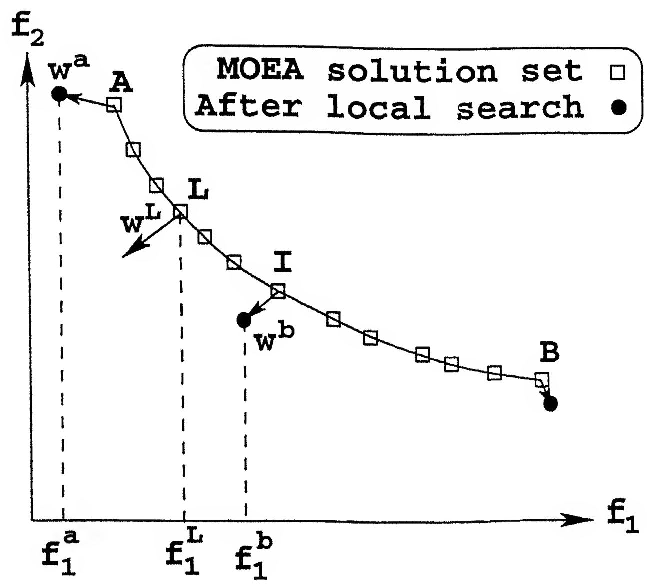

Figure 4 5 The continuously-updated weight approach is illustrated

First, the extreme solutions A and B are assigned a weight vector according to equation 4 4 A local search procedure is used to find the corresponding optimum solutions Based on the new values of f_j^{min} and f_j^{max} values, an intermediate solution I is assigned a weight vector Preferably, this intermediate solution I can be chosen as the one having maximally away from the extreme solutions The local search procedure can be used with this intermediate solution and the optimum solution can be found Now

each of the two intervals are considered separately region between A and I and the region between I and A An intermediate solution (preferably the solution in the middle of the range) is chosen in each range Say the solution L is chosen in the range between A and I The pseudo-weight vector for this solution is calculated as follows

$$\bar{w}_j^{L} = \bar{w}_j^{a}\frac{f_j^{b} - f_j^{L}}{f_j^{b} - f_j^{a}} + \bar{w}_j^{b}\frac{f_j^{L} - f_j^{a}}{f_j^{b} - f_j^{a}}, \tag{4 5}$$

where solutions a and b are extreme solutions for the range under consideration Thus for solution L solution a is A and solution b is I The parameters $\bar{w}_j^{a}$ and $\bar{w}_j^{b}$ are pseudo-weights associated with the extreme solutions When pseudo-weights are calculated for objective functions, they can be normalized This procedure can be repeated till the local search is applied to all solutions This update of pseudo-weights with a local search procedure is advantageous in three ways -

1 The true optimum solution corresponding to a weight vector can be found, and

2 The calculated weights for each optimum solution is close to their true weights

3 This gives sufficient space to spread the solutions, which are biased towards one region i e the Pareto-optimal front is rearranged according to the bias towards some region

The detailed methodology is given here

1 Take the extremities of any function and assign the weights as giving importance to that function only and ignoring others Assign weight value unity to the corresponding best function value Use the fact $\sum_{i=1}^{M} \bar{w}_i = 1$

2 Find new extremities after hill climbing

3 Set weight of the extremities as bounds on the weights

4 To find the approximate middle point of the region bounded by the weights, weight of all the individual between the region is find by using formula

$$w_j^{i} = w_{2j} + \frac{(w_{1j} - w_{2j}) * (f_{2j} - f_j^{i})}{(f_{2j} - f_{1j})}$$

where

w_j^i	j_{th} weight of the i_{th} individual
w_{2j}	Lower weight bound to the j_{th} fitness function
w_{1j}	Upper weight bound to the j_{th} fitness function
f_{2j}	Scaled value of the j_{th} objective function corresponding to weight w_{2j}
f_{1j}	Scaled value of the j_{th} objective function corresponding to weight w_{1j}
f_j^i	Scaled value of j_{th} objective function of the i_{th} individual

5 Do hill climbing on the individual which is approximately in the middle of the given bounds and find the pseudo-weight $\bar{w}$ using equation 4 5

6 Now split the region into two parts [1] One between first bound and the middle point and other between the middle point and second bound Goto Step 4 and do the hill climbing recursively until all the individuals are taken care

This formula of weight calculation ensures that the weights are normalized or $\sum_{j=1}^{M} \bar{w}_j' = 1$

.6 5 Clustering

he number of solutions obtained just after the non-domination search on the solutions obtained by cal search may be more than required To reduce the number of solutions to the desired number ie clustering approach is used The approach used for the clustering is similar to that used in [36] for ducing the size of non-dominated set The Figure 4 6 is an illustrative sketch of the clustering Here itially all N solutions are initially assumed to belong to a separate cluster Thereafter the centroid all the different clusters and then euclidean distance d_c between all these different cluster pairs is lculated Two different clusters having the minimum distance are merged to form the bigger cluster his procedure is repeated until the number of different clusters is reduced to the number of solutions quired by the user Finally, from each cluster the solution closest to the centroid is selected Rest all ie solutions are dropped This is how the cardinality of the solution set is reduced Figure 4 6 shows ie non-dominated solution set in the open boxes and the reduced set in solid boxes

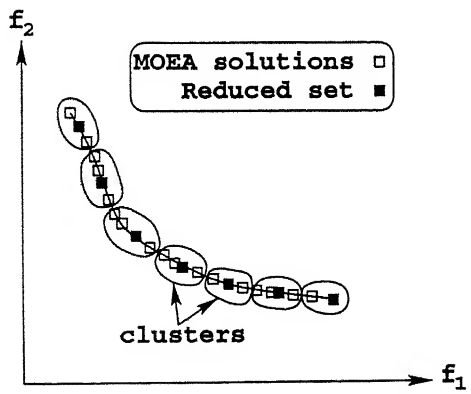

Figure 4 6 The clustering method to reduce the non-dominated solutions set

[1] The region is split according to the function values before hill climbing Because after hill climbing the individual elected may move to one side and include or exclude few individuals which were previously inside the region of one bound nd this approx middle point but now moved to the middle point and other bound This shifting from one side to other ay cause some error in finding natural weights

4.7 Closure

The multi-objective optimization problems have more than one objective which are to be satisfied simultaneously. The problem becomes complex if the objectives are conflicting in nature. The solution to these problems is a set of solutions called Pareto-optimal sets. Different multi-objective optimization algorithms are suggested; a specific MOGA—elitist non dominated sorting genetic algorithm (NSGA-II) is selected. The dominance definition used for ranking the population is changed to handle the constraints. Each solution of the Pareto-optimal set obtained by NSGA-II undergo a hill climbing local search and the new solution is obtained. This new solution set gives a new Pareto- optimal front after the non domination search is carried out for eliminating the dominated solutions. It is expected to have a better convergence than before. The size of the non-dominated solution set is also expected to be reduced as a few non-dominated solutions can converge to the same optima. The local search uses a single objective function. The multiple objectives are converted into a single objective by using either the fixed weight strategy or by using the continuous updated weight strategy. The number of solutions is further reduced to a user specified number by the clustering method.

Chapter 5

Multi-objective problems and the results

In the third chapter, the applicability of the hybrid approach to the solve the single objective shape design problems has been discussed It is found that the solutions obtained by considering single objective at a time is not sufficient, as with a change in random number the optimum function value does not change much, but some times the appearance changes very much This also bring changes in stresses as well In this chapter the study is extended to the most important part of the study application of hybrid approach to the multiple objectives problems It means same problem is formulated as *Multi-objective optimization* problem by bringing more objectives To solve this set of problems, from the set of different multi-objective genetic algorithms (MOGA), Non-dominated sorting genetic algorithm (NSGA-II) is selected This algorithm can maintain diversity on the front This feature is exploited to find a number of diverse solutions The given solution set is then refined using a local search to find the optimal solution On top of this, a clustering algorithm is used to reduce the number of available solutions to a desired number This helps the designer find as many different solutions as he wants and then selection can be done according to his requirement The use of genetic algorithms help the designer in finding new shape which increases the creativity in the designs This has the biggest advantage of giving the multiple results in just one shot

For this study all the problems which were considered for single objective case are solved for two objectives The problem of design of cross-section does not involve any loading so the weight of the design does not affect the design All these problems other than the problem of design of cross-section, are solved for two different cases

1 When the weight of the design is not considered

2 When the weight of the design is also taken into consideration while calculation of the loading

For the problem of the design of the cross-section, moment of inertia to weight ratio for two axis, is maximized by assigning one objective function to each axis All other problems solutions are solved for

following two objectives

- First objective is minimization of weight
- Second objective is minimization of 1/Stiffness Since stiffness is given as inverse of displacement so this objective can be redefined as minimization of displacements

For all the test problems considered, other than the design of cross-section, following GA parameters are selected

Population Size	30
Crossover Probability	0 95
Mutation Probability	1/String length
No of Generations	150

For the design of cross-section the only difference is that the population size is taken to be 100 instead of the 30

The description of the problems is given in the third chapter The change in the solution is that the maximum displacement is also treated as an objective here The deflection is scaled such that the order of both objective values becomes the same This objective is also a constrained objective

5.1 Design of the cross-section

The problem is described in the chapter 3 The results for the single objective optimization are also presented in the chapter 3 The results obtained for these problems for the single objective case clearly show that the approach considered has the potential of finding the optimal solution The next and more important step in solving the problem is to consider more than one objectives This problem is solved for two objectives- maximization of moment of inertia to the weight ratio for two axis

First plot in the Figure 5 1 shows all the non-dominated solutions obtained after the NSGA-II simulation runs Since the population size is 100, almost 100 non-dominated solutions are obtained Second plot shows the solutions obtained after the local search is carried out on the solutions set obtained after the NSGA-II simulation runs This is clear from the plot that the cardinality of the solution is reduced The non-domination local search on this set of the solutions gives the final non-dominated set as presented in the third plot The number of the solutions present in this set is 81 The graph shows that the diversity is lost on the Pareto-optimal front The solutions which are maximally away from the ends, mostly give way to the solutions near to the optima for one objective functions Here some solutions have converged to one particular solution after the local search because the weight for

each solution is determined individually, without considering the effect of the others That is also the reason why some dominated solutions are obtained after the local search Out of these 81 solutions nine solutions are selected by means of the clustering These are shown in the last plot That's how the final selection of the solutions is made

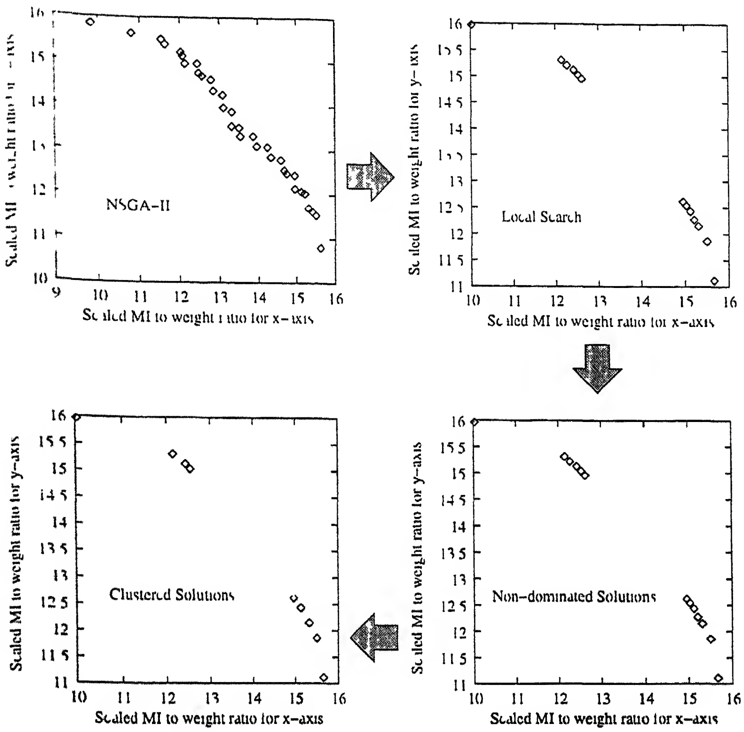

Figure 5 1 Hybrid procedure to find nine trade-off solutions for the cross-section design problem

In order to visualize the obtained set of nine solutions having a wide range of trade-offs in the scaled moment of inertia (MI) to weight ratio for both the axis, the shapes in Figure 5 2 are presented It is clear that starting from a solution with large MI to weight ratio for one axis, how the shapes for the large MI to weight ratio for other axis are obtained by the use of the hybrid approach The solutions are presented in the order of increasing MI to weight ratio for one axis, in the form of a 3 × 3 matrix The first solution at position (1,1) in the matrix, has the maximum MI to weight ratio for x-axis The solution places the vertical bar on ends to increase the moment of inertia for other axis, i e increase in MI to weight ratio for other axis The solutions (1,2) and (1,3) increases the ratio for y-axis by putting more material far from the center Sixth solution (2,3) of the matrix, gives more importance to the moment of inertia to weight ratio for the y-axis The solution has lesser moment of inertia to weight ratio for x-axis The ninth solution (3,3) is having the maximum MI to weight ratio for y-axis It is clear from the solutions that it tries to keep more material far from the axis to increase the desired ratio This also tells the method of placing material to increase the moment of inertia in a optimal way

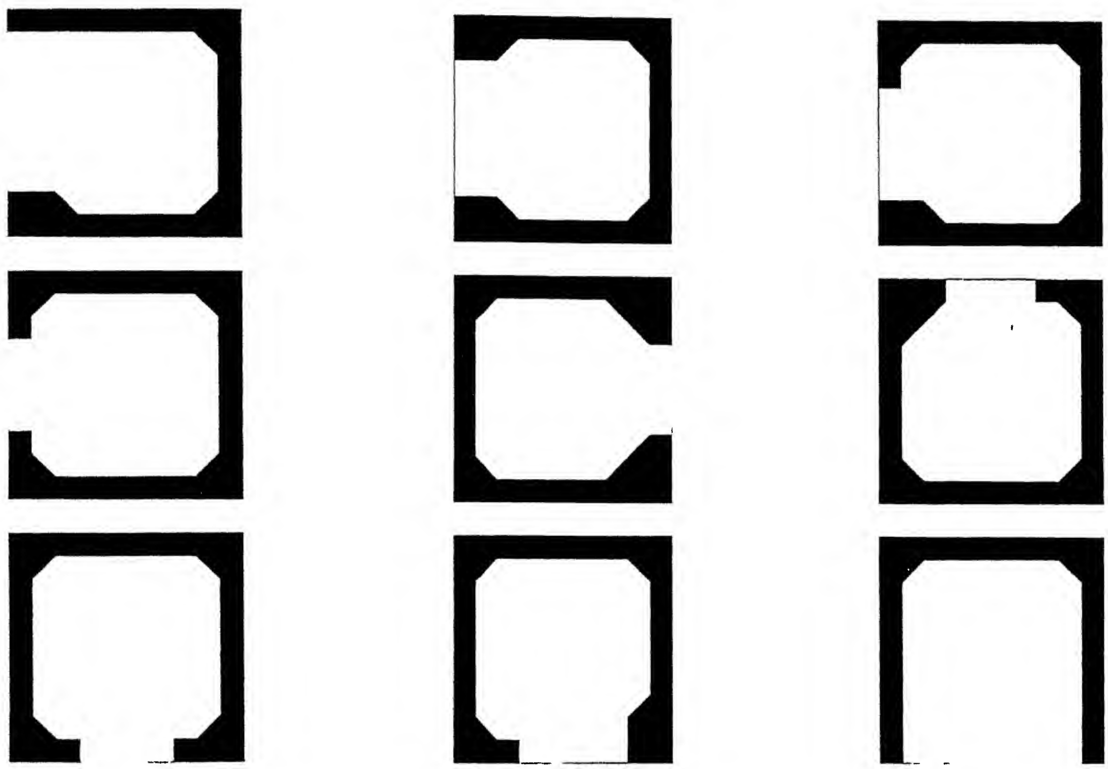

Figure 5 2 Nine trade-off shapes for the design of the cross-section

5.2 Design of the cantilever plate

The loading conditions and the supports for the cantilever plate are shown in Figure 3 1 The results for the cases when the weight of the plate is considered to have some effect on the loading and when the weight of the designed plate is not considered are presented separately

5 2 1 Design of the plate when its weight is not considered

Figure 5 3 presents the four steps of the proposed hybrid method applied on this problem First plot shows the non-dominated solutions obtained using NSGA-II Since the population size is 30 NSGA-II is able to find 30 different non-dominated solutions Thereafter, the local search method is applied from each non-dominated solution and new and improved set of solutions are obtained The third plot is the result of the non-dominated check of the solutions obtained after the local search method Three dominated solutions are eliminated by this process The final plot is obtained after the clustering operation with a choice of nine solutions The plot shows how nine well distributed set of solutions are found from the third plot of 27 solutions If fewer than nine solutions are desired, the clustering mechanism can be set accordingly

In order to visualize the obtained set of nine solutions having a wide range of trade-offs in the weight and scaled deflection values, the shapes in Figure 5 4 are shown It is clear that starting from a low-weight solution (with large deflection), how large-weight (with small deflection) shapes are found by the hybrid method

It is interesting to note that the minimum weight solution eliminated one complete row (the bottom-most row) in order to reduce the overall weight The second solution (the element (1,2) in the above 3×3 matrix) corresponds to the second-best weight solution It is well known that for an end load cantilever plate, a parabolic shape is optimal Both shapes (elements (1,1) and (1,2)) exhibits a similar shape As the importance of deflection increases, the shapes tend to have more and more elements, thereby making the plate rigid enough to have smaller deflection In the middle, the development of vertical stiffener is interesting This is a compromise between the minimum weight solution and a minimum deflection solution By adding a stiffener the weight of the structure does not increase much, whereas the stiffness of plate increases (hence the deflection reduces) Finally, the complete plate with right top and bottom ends chopped off is the minimum deflection solution

All nine solutions (and if needed, more can also be obtained) with interesting trade-offs between weight and deflection are obtained in one simulation run of the hybrid method More importantly the solutions to this problem present the way of putting material to reduce the deflection in an optimal way

5 2 2 Design of cantilever plate when weight is also considered

Now the weight of the designed plate is also considered to effect the loading Figure 5 5 shows the non-dominated solutions obtained using the hybrid approach A careful look on the solutions for the case when weight is considered and when weight is not considered clearly tells the effect of the weight consideration The high density increases the load so the solution spread is reduced on both axis i e the range of the weight and the range of the deflection is reduced The NSGA-II gives 30 non-dominated solutions The local search takes the solutions to a better distribution The local search improves the diversity among the solutions The number of the solutions is reduced to 20 from 30 after the non-domination check is carried out on the set of the solutions obtained after local search Clustering mechanism is used to extract the nine diversified solutions from this non-dominated set These nine solutions are shown in the Figure 5 6

The solutions are arranged in the order of the increasing weight The minimum weight solution (left solution in the top most row), exhibits the shape very similar to the parabolic shape The solution has two elements joining the node bearing the load to the rest of the plate Next solution element (1 2) has more symmetric shape Another interesting shape is the third solution (shown in the (1 3)th position of the matrix) This solution has a vertical stiffener besides the parabolic shape This gives it additional strength without increasing the weight much Other solutions increases the material near to the stiffener to increase the strength and hence reducing the deflection Finally the solution presented in the position (3,3) of the matrix is for the maximum weight Here the number of stiffeners has also increased so the deflection is reduced This solution is significantly different than the maximum weight solution obtained in the previous case Most interesting point of this solution is the number of holes The number of holes is increased due to more number of stiffeners This help in reducing the weights while keeping the deflection minimum

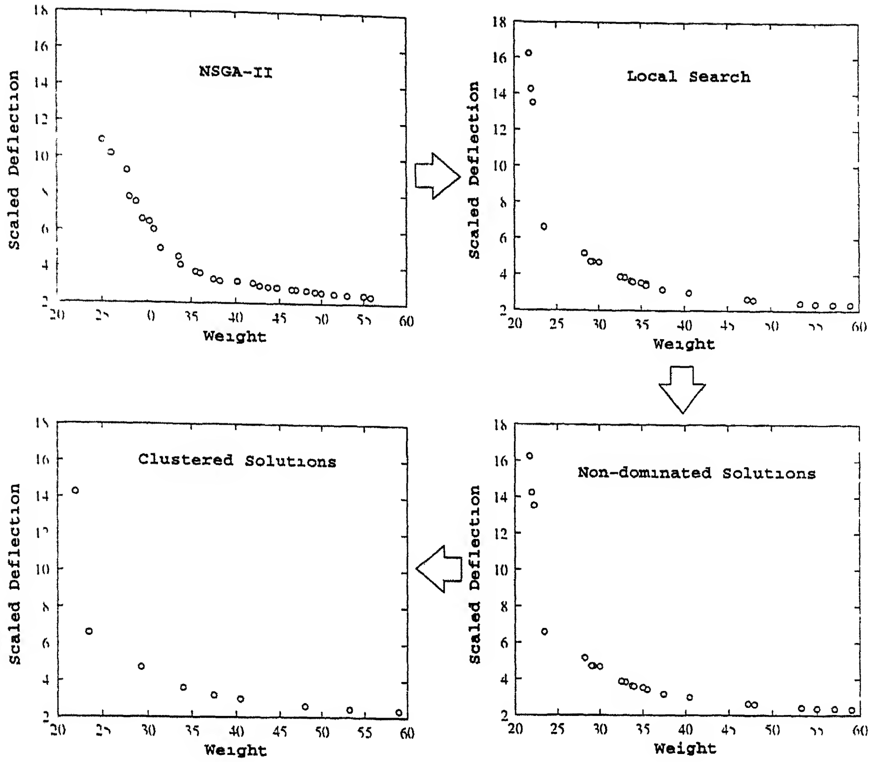

Figure 5 3 Hybrid procedure to find nine trade-off solutions for the cantilever plate design problem

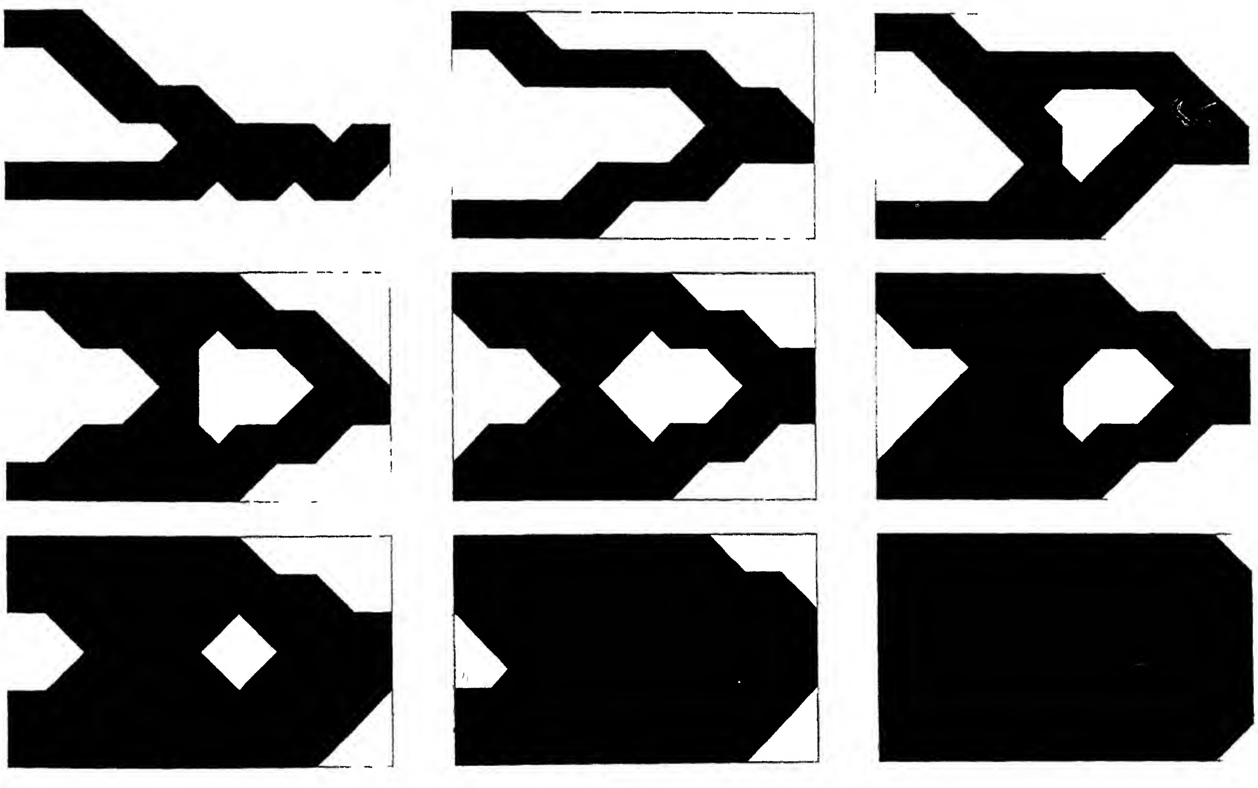

Figure 5 4 Nine trade-off shapes for the cantilever plate design

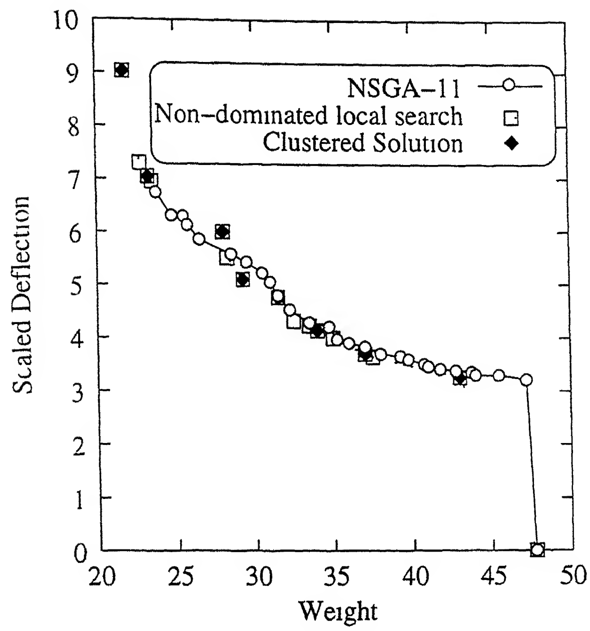

Figure 5 5 Hybrid procedure to find nine trade-off solutions for the cantilever plate design problem when weight is also considered

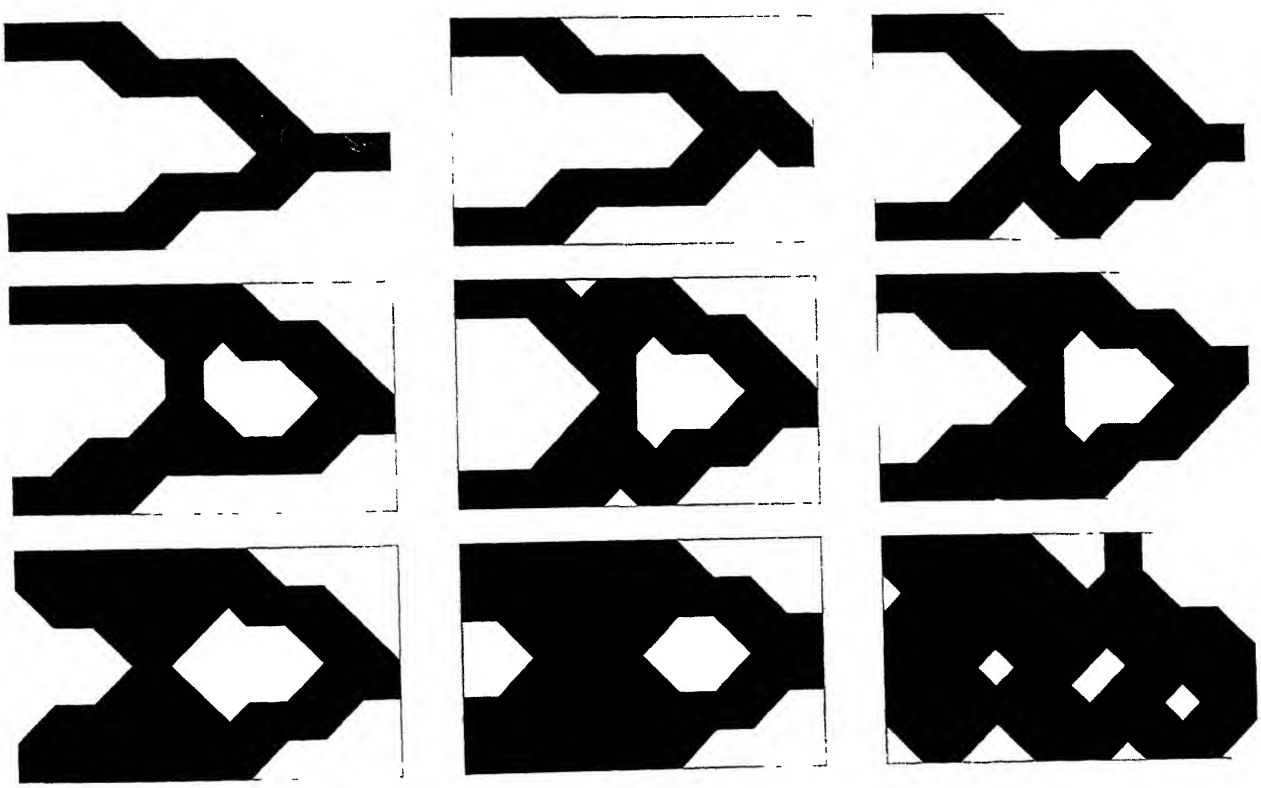

Figure 5 6 Nine trade-off shapes for the cantilever plate design when weight is considered

5.3 Design of the simple supported plate when point load is applied on the topside

Next problem is the design of the simple supported plate when a point load is applied on the topside of the plate as shown in the Figure 3 2 The results for the cases when the load of the design itself is accounted for the calculation of the weight, and the case when load is not accounted as loading, are presented

5 3 1 When weight of the designed plate is not considered

Figure 5 7 shows the hybrid approach to find the nine solutions The NSGA-II obtains the non-dominated set of the solutions These solutions are further improved by the local search giving a better distribution on the Pareto-optimal front The shape of the Pareto-optimal front is also improved in this exercise of carrying out the local search The number of solutions is reduced to 22 after carrying out a non-domination check on the solution set obtained by the local search The method reduces the cardinality of the problem to some extent Nine solutions are selected from this set using the clustering approach These solutions are presented in the Figure 5 8

An analysis of the solution reveals some interesting information The minimum weight solution (1 1)th in the matrix, tends to use one row less but since the load is applied in the top row, so in-order to carry the load one element is placed in the top row The third solution is an interesting solution A careful look of Figure 5 7 reveals that this solution is a knee solution It means the solution makes a large sacrifice in the deflection to achieve a small advantage in the weight Similarly, to achieve a small advantage in deflection-loss, a large sacrifice in weight is needed Shapes in position (1,2) and (2,1) can be compared with respect to the shape in position (1,3) Shape in position (3,1) or solution 7 is also interesting In order to have further reduction in deflection stiffening of the two slanted arms is needed These stiffener of the slanted arms form a close loop or it can be said that the solution has got 3 holes Solution (3,2) or 8th solution has thickened the slanted arms, but one hole is there to reduce the weight Finally, the absolute minimum deflection shape is the complete rectangle with maximum possible weight One very important point to note here is the almost symmetric nature of the shapes

Starting with the minimum weight design having two slim slanted legs down to thickening the legs to make them stiff, followed by joining the legs with a stiffener, and finally finding the complete rectangular plate having minimum deflection are all intuitive trade-off solutions In the absence of any such knowledge, it is interesting how the hybrid procedure with NSGA-II is able to find the whole family of different trade-off solutions

5.3.2 Weight of the plate is also considered.

Now the same plate is designed when the weight of the design is also considered while calculation of the stresses and deflection. Figure 5.9 shows the solutions obtained by the NSGA-II simulations. Local search on these solutions increases the diversity. The non-domination check gives 29 non-dominated solutions after the local search. Here the number of solutions is not reduced very much but the spread of the solutions is improved. The solution set is reduced to nine by clustering. The final nine solutions are presented in the form of a matrix of 3 × 3 in Figure 5.10.

A close look on the Figure 5.7 and Figure 5.9 reveals the fact that the diversity of the solution set is reduced when the weight is also taken into consideration. Now the spread of the front is reduced to 15-40 units instead of 15-60 units on the weight axis and the spread is also reduced on the scaled deflection axis. The shape of the Pareto-optimal front is evident of the improvement in the solutions by the local search. The solution in the first row is the same as the solutions when the weight is not considered. The solutions in the second row i.e. solutions (2,1) and (2,2) put more material on the arms. Since the weight of the design is also taken as the distributed loading so the material is also put in the optimal way such that the effect is not to increase the deflection. The material is put near to the supports such that it has lesser effect due to the weight of the designed plate. The solution at the position (2,3) seem very interesting as this solution puts same amount of the material everywhere. Rest all other solutions in the last row of the shape matrix seem a derivative of this solution. All the solutions in the last row have lesser deflection and more weight, but unlike the case when weight is not considered, the designs put lesser material in the design and the reason is very obvious. Here more material means more stress and more deflection, hence lesser material is used. So the solution with maximum amount of the material is not the same as the whole plate. The solution with the maximum weight is also now having the deep chopped corners with thick arms. Interestingly, all the solutions are symmetric in nature and no hole is present in any solution.

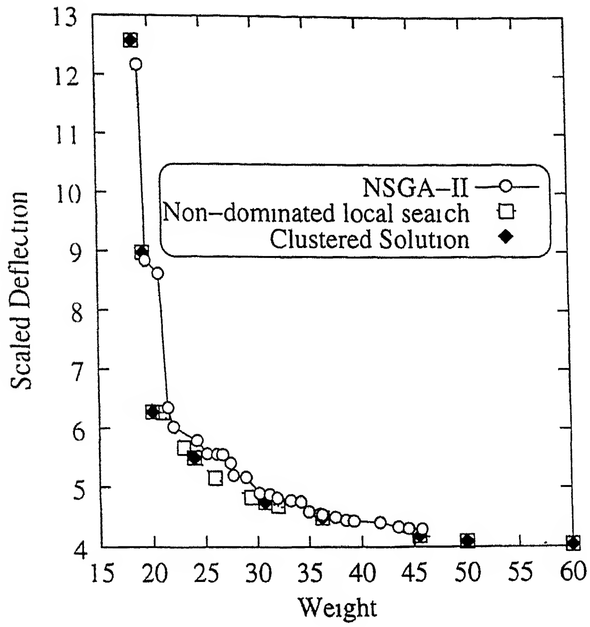

Figure 5 7 Hybrid procedure to find nine trade-off solutions for the simple supported plate design problem when weight is not considered

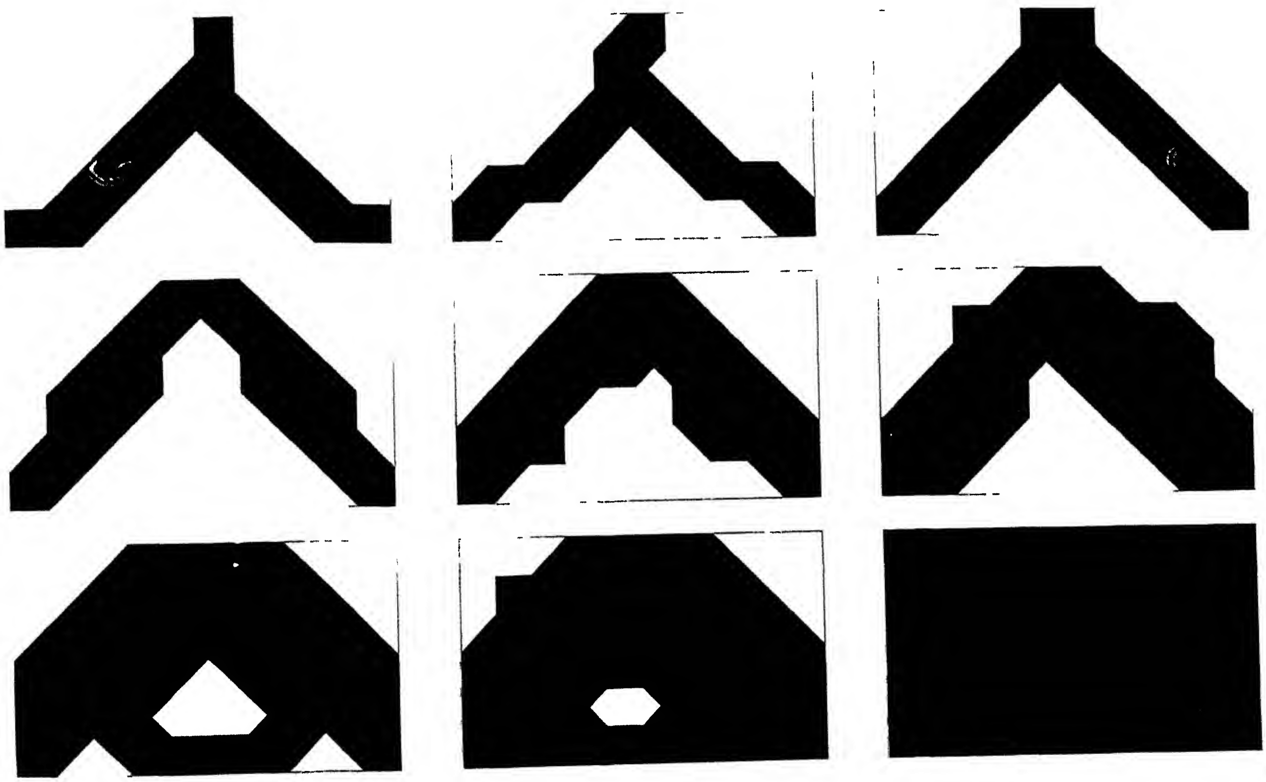

Figure 5 8 Nine trade-off shapes for the design of simple supported plate with a point load applied on the top

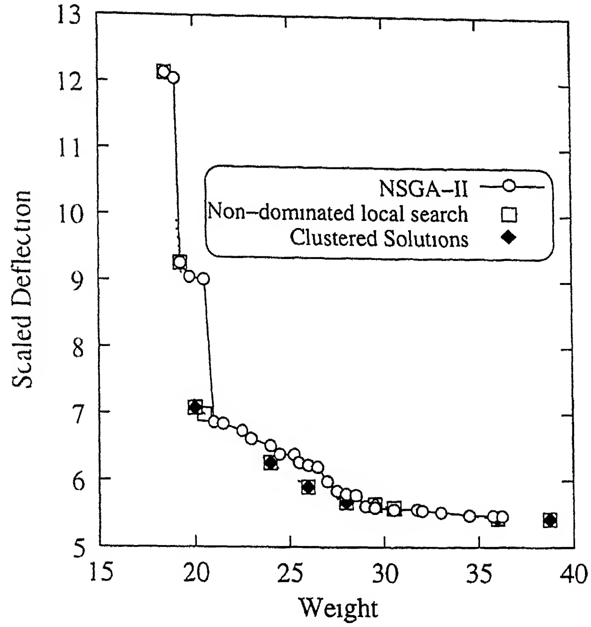

Figure 5 9 Hybrid procedure to find nine trade-off solutions for the simple supported plate design problem when weight of the plate is considered

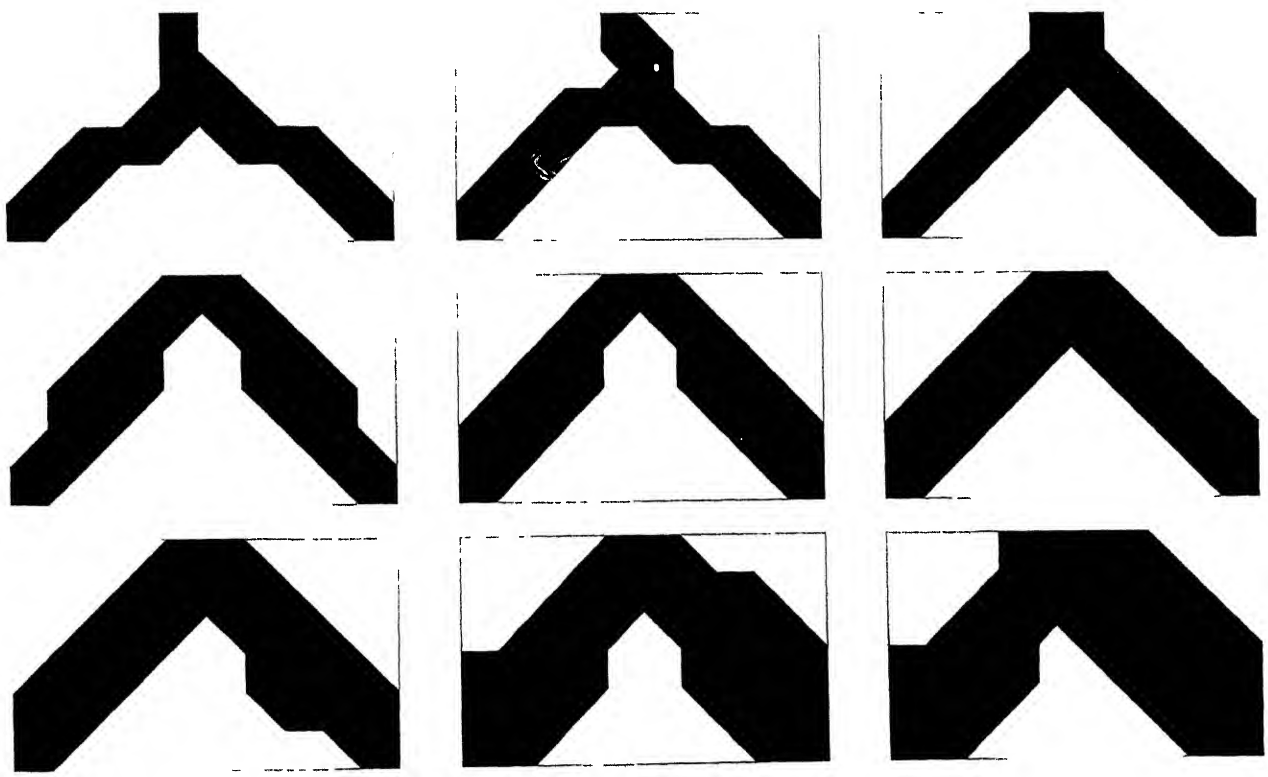

Figure 5 10 Nine trade-off shapes for the design of simple supported plate with a point load applied on the top and the weight is design also considered

5.4 Design of the simple supported plate with a distributed load applied on the top

The supports and the loading conditions of the problem are shown in Figure 3 3 The problem is such that it requires all the elements in the first row to be present to make the design geometrically feasible The results for both of the cases are presented in the next subsections

5 4 1 Weight of the plate is not considered

First the results for the case, when weight of the design is not considered, are presented in Figure 5 11 The solutions obtained by the NSGA-II are presented here The Pareto-optimal front obtained by NSGA-II runs have a step in the front Local search improves the spread of the solution set and the convex Pareto-optimal front is obtained after the local search The spread on the weight front is improved from 26-53 units to 22-60 units and on the scaled deflection the improvement is clearly visible as the result of local search The non-domination check on this new solution set, yield 21 well spaced non-dominated solutions This reduces the cardinality of the problem and take the approach towards more practical side Nine diverse solution are selected through clustering and are presented in the Figure 5 12

The solution with the least weight is solution (1,1) of the matrix This solution has the straight arm joining the left end support with the top load carrying part of the plate The right arm has got a small twist at the joining point The second solution at (1,2)th position is a symmetric solution which shifts the joining points of the supports with the load carrying arm towards inside giving a shape of the table From the solutions shown in Figure 5 11, it is found that this solution is a knee solution A big compromise is made on the deflection, to accommodate more weight The solution (1,3) and (2 1) keep the symmetry but thicken the arms The trend of keeping more material on the supports continues such that the tabular structure is obtained and the supports are joined with the top row elements through the straight arms The small shift in the symmetry towards one side is just due to the attempt of the search of the solutions with different stiffness The maximum weight solution is the whole plate (the maximum possible weight)

5.4.2 Weight of the designed plate is considered

Figure 5 13 shows the result obtained by using the hybrid approach to solve the problem Initially, NSGA-II is used to find the solution set of the problem The nature of the Pareto-optimal Front obtained by NSGA-II gives an expression that the some part of front is concave The solutions are improved by the local search The Pareto-optimal front is improved by the local search and the front is found to be convex with the presence of a few steps Only 17 non-dominated solutions are obtained

by this process of local search and the non-domination search Actual number of different solutions is found to be nine Thus the number of solutions is reduced fairly The solution set is less diverse than the case when weight of the design was not taken care The weight axis diversity has reduced to 20-50 from 20-60 The diversity on the deflection axis has undergone a major change as it is reduced to 1 6-3 2 from 1 0-8 0 in the previous case The reduction in the number of different solutions may be the presence of optimal points in the basin where the different solutions are taken by the local search Thus no control on spread can be obtained by this weight selection strategy Besides this the weights for the solutions in the concave region may not be very appropriate This is the limitation of the weighted sum strategy of not reaching the optimal weights in the concave region The solutions which are very near to each other also converge to one solution That's why the diversity after the local search is affected Nine different solutions are presented in Figure 5 14

The minimum weight solution (position (1,1) solution) is symmetric and has the got the tabular shape Other solutions like those shown in (1,2) and (1,3) position of the 3 × 3 matrix have more material placed on the arms The third solution is a knee solution as a small change in the weight has caused a large change in the deflection values The solutions in the second row are very near to each other in the fitness space and therefore have very much similarity in the shapes Solutions in the last row restore diversity in the fitness values and as well seen in the shapes More weight is put on the arms such that the joints are thickened The tendency is to keep the weight or the material near to the supports thus reducing the effect of the increase in weight This way the solutions does not have got very much increase in the deflection The maximum weight solution have thick symmetrically placed arms All the solutions are almost symmetric and have a near tabular shape with thick slanted arms

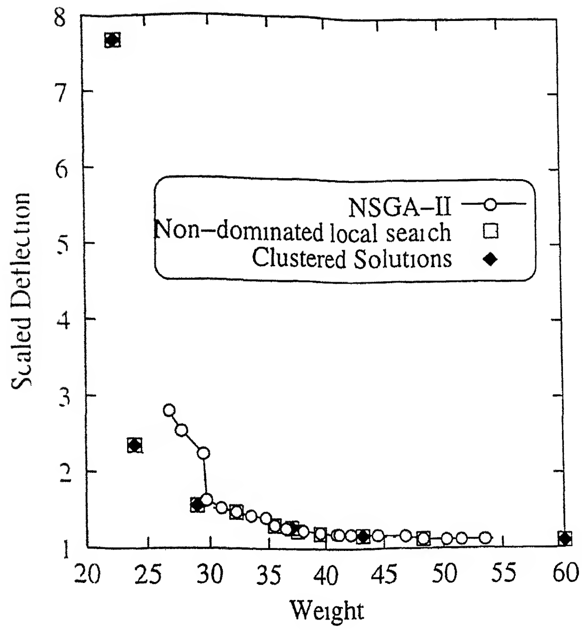

Figure 5 11 Hybrid procedure to find nine trade-off solutions for the simple supported plate design problem with a distributed load on the top and the weight of the design is not considered

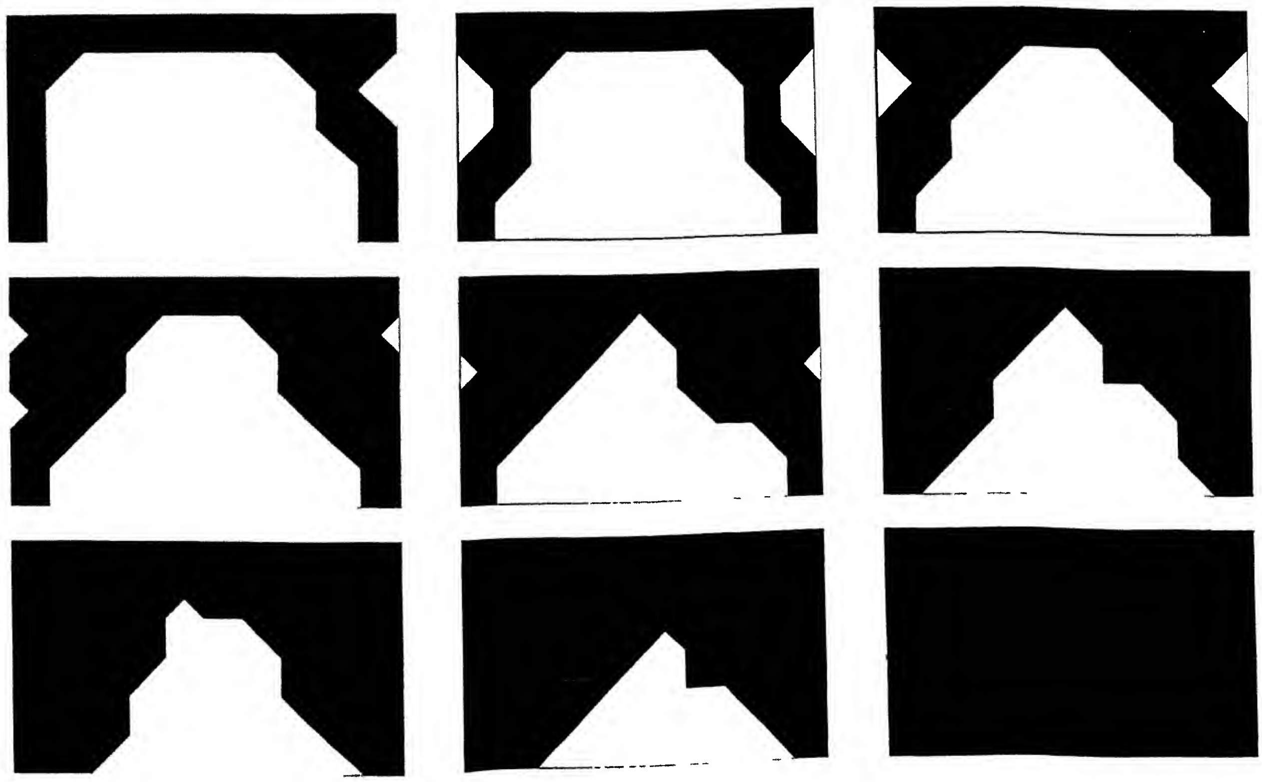

Figure 5 12 Nine trade-off shapes for the design of a simple supported plate witha distributed load applied on the top

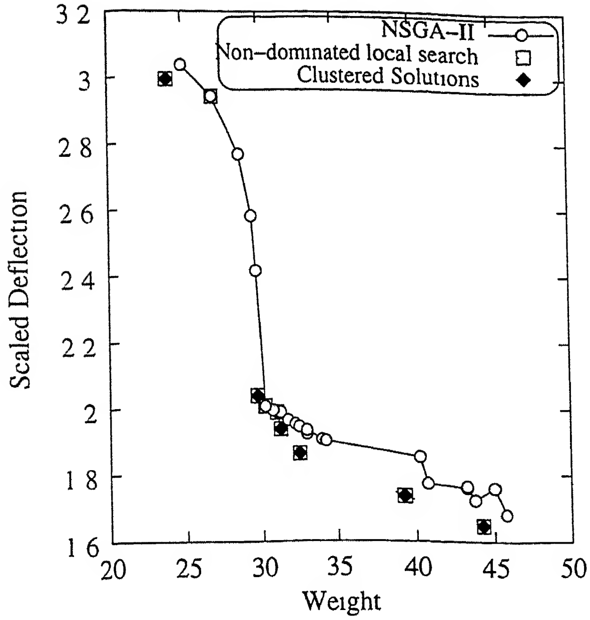

Figure 5 13 Hybrid procedure to find nine trade-off solutions for the simple supported plate design problem with distributed load and weight is also considered

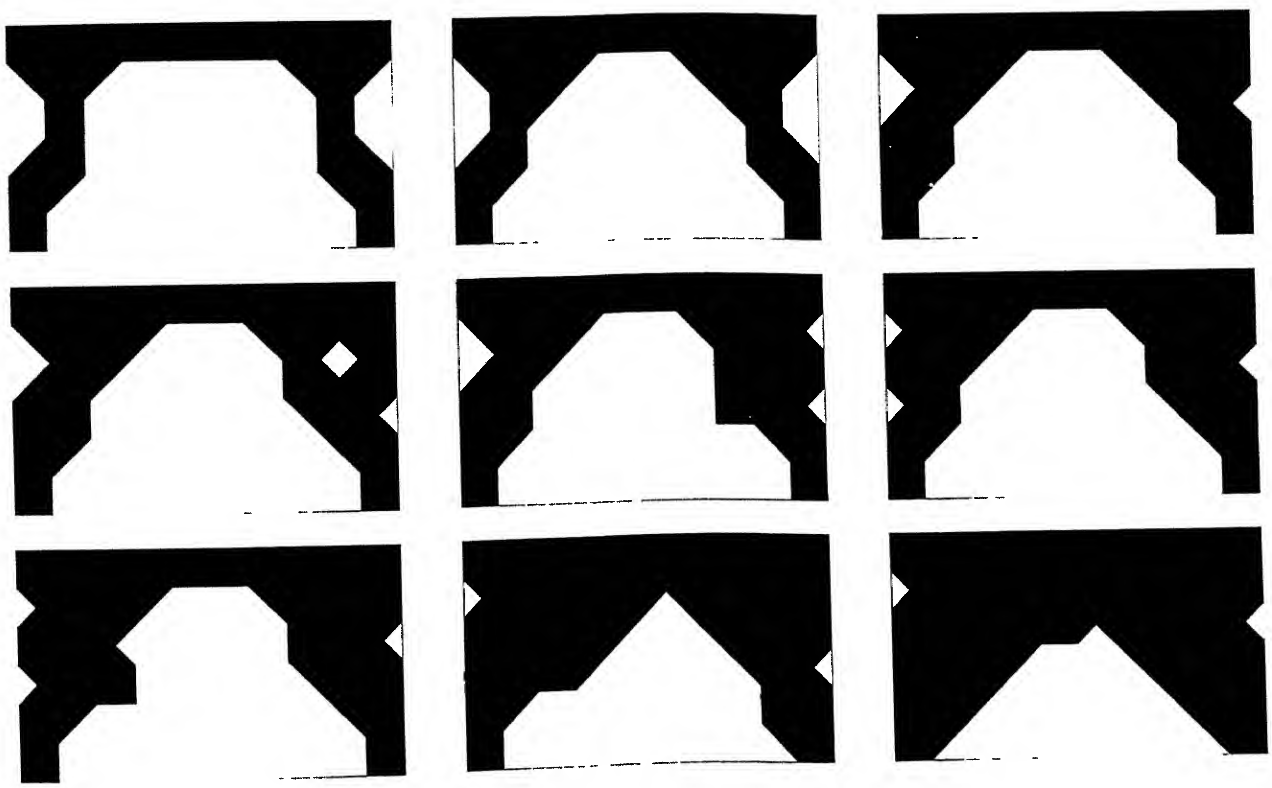

Figure 5 14 Nine trade-off shapes for the design of simple supported plate with a distributed load on the bottom side when the weight of the design is also considered.

5.5 Design of the simple supported plate when a point load is applied on the bottom side

The loading and the supports for this problem is shown in the Figure 3 4 Now the loading is applied on the lower side of the plate, which makes it feasible to eliminate the top portion, if not required The same trend is observed in the solutions for both of the cases, when weight of the design is considered and when weight of the design is not considered

5 5 1 Weight of the designed plate is not considered

The results for this problem is shown in Figure 5 15 Initial solution set of 30 non-dominated solutions is obtained by the NSGA-II For low weight the solution set obtained is having the front which seem to have some steps The solution set has a few cluster of the solutions The local search on this solution set further improves the diversity and the shape of the Pareto-optimal front is also found to be convex without steps Despite the clusters of the solutions in the initial solution set, the new set after the local search has solution spread at intervals on all over the front and 28 non dominated solutions are obtained after the non domination check Out of these 28 solutions only 13 are different solutions The spread of the solutions on the weight axis is from 10 units to 60 units Similarly, on the deflection axis, the spread is from 2 0 to 14 0 units Nine different solutions are taken from the set of 13 different solutions and presented in Figure 5 16

The solutions obtained here are very interesting The minimum weight solution at position (1,1) in the matrix eliminates the three rows completely The solution is very interesting in the sense that this joins the ends in such a manner that there are two holes These holes in the structure reduce the weight The third solution is also interesting The solution is symmetric and is placed such that the material is placed more in the middle of the plate where the load is applied This solution follows the shape of the bending moment diagram for this problem More material is placed in the zone of higher bending moment The solution is also interesting in the manner that this solution requires very much reduction in the deflection to optimally increase the weight by a small amount as shown by the slope of the line joining this solution and the previous solution The fourth solution at position (2,1) has put more material in the middle of the plate near to the load This helps in reducing the stress by the increase in the cross-section area on this section Similarly, the fifth solution (2,2) is also symmetric and have more material near the load The arms are also thickened now This solution has made use of one more row but the material is placed symmetrically Interestingly, all these solutions maintain two cavities and try to maintain the symmetricity in the shapes The solution for the maximum weight have whole of the plate as the solution

5 5 2 Weight of the designed plate is considered for the calculation of the loading

Now the case when the weight of the designed plate is also considered for the calculation of the stresses and deflection is analysed The results are presented in the Figure 5 17 The solution set obtained by the NSGA-II gives reasonable diversity but the shape of Pareto-optimal front is not smooth The local search improves the solutions such that the resulting solution set gives a Pareto-optimal set without steps and the diversity on the front is improved 29 non dominated solutions are obtained from the non-domination search on the set obtained after the local search, out of which only 15 are different in shape The spread of the solution set is slightly lesser than the case when the weight of the designed plate is not considered for calculating the load, as the loading increases with the increase in the weight Nine solution which are sufficiently distinct in the objective function space are selected and presented in the Figure 5 18

The minimum weight solution is almost the same as in the case, when the weight of the plate is not accounted for calculation of the loading Solution (1,2) and (1,3) put more material in the middle of the plate But they are no more symmetric The solutions has a tendency to increase the cross-section just above the load to reduce the stresses and the deflection The fourth solution (2,1) seem very interesting as this has put the maximum material in the middle just above the load The material experience a pull towards down in the middle and the slanted arms restricts the extent of the pull Other solutions are such that they use more material to stiffen the slanted arms This make it possible to reduce the deflection But the cavities on the bottom side are retained This shows that the material at the place of cavities can be safely taken away to reduce the weight The maximum weight solution is also having the cavities and remove the material on the top ends Interestingly, all the solutions are nearly symmetric in nature and the material is placed according to the variation in the bending moment

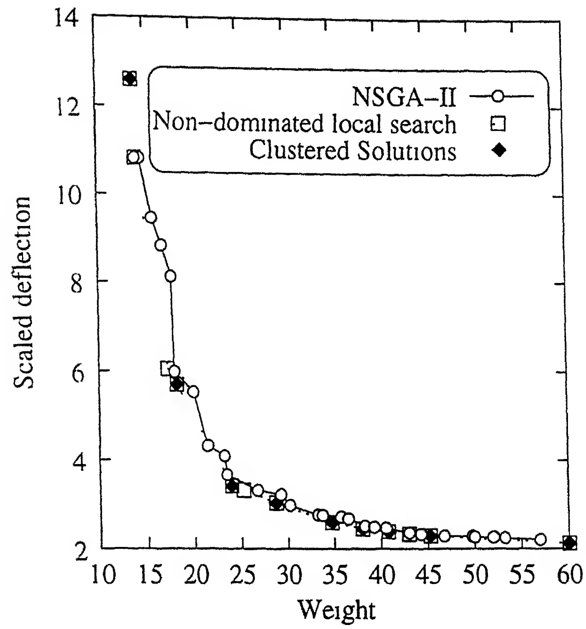

Figure 5 15 Hybrid procedure to find nine trade-off solutions for the simple supported plate design problem when point load is applied on the bottom side and weight of the design is not considered

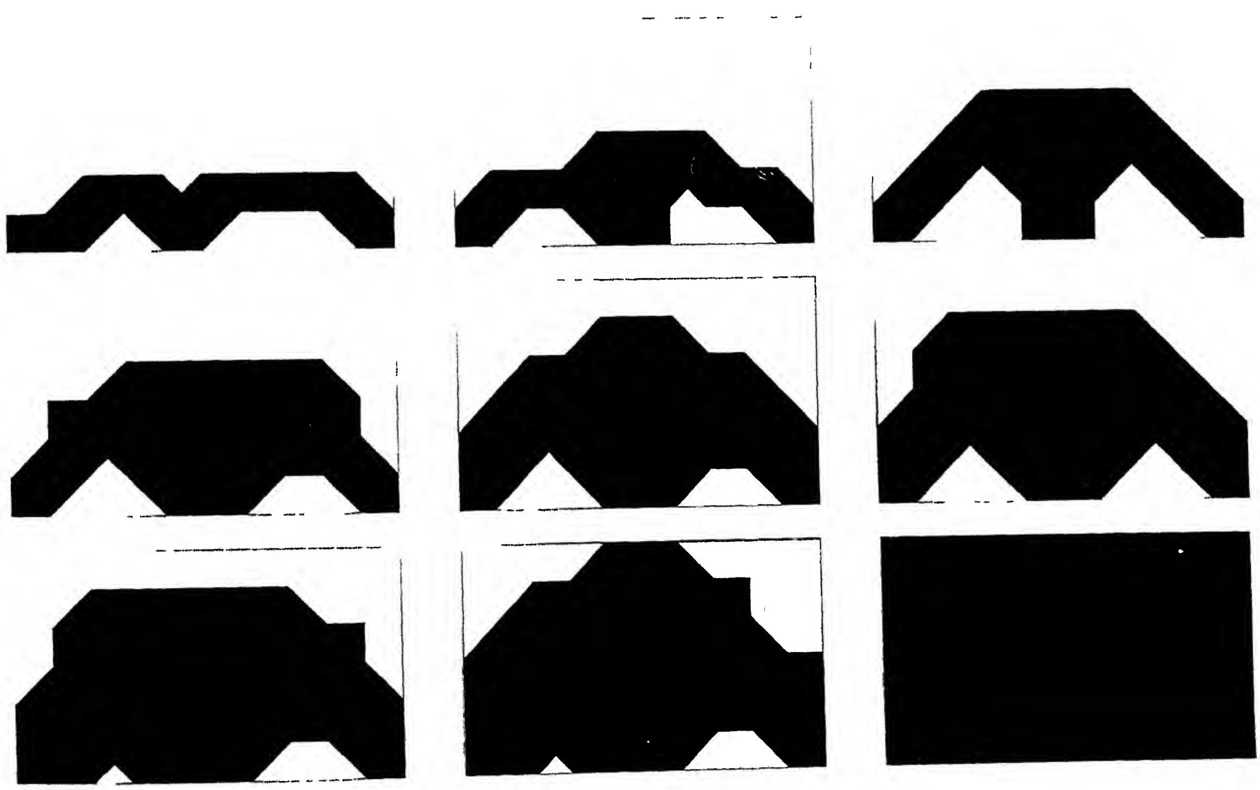

Figure 5 16 Nine trade-off shapes for the design of simple supported plate with a point load applied on the bottom side

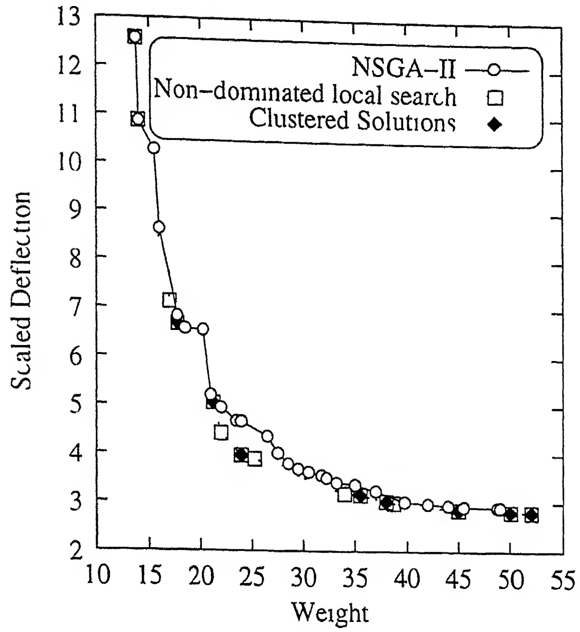

Figure 5 17 Hybrid procedure to find nine trade-off solutions for the simple supported plate design problem with a point load applied on the bottom side and weight consideration

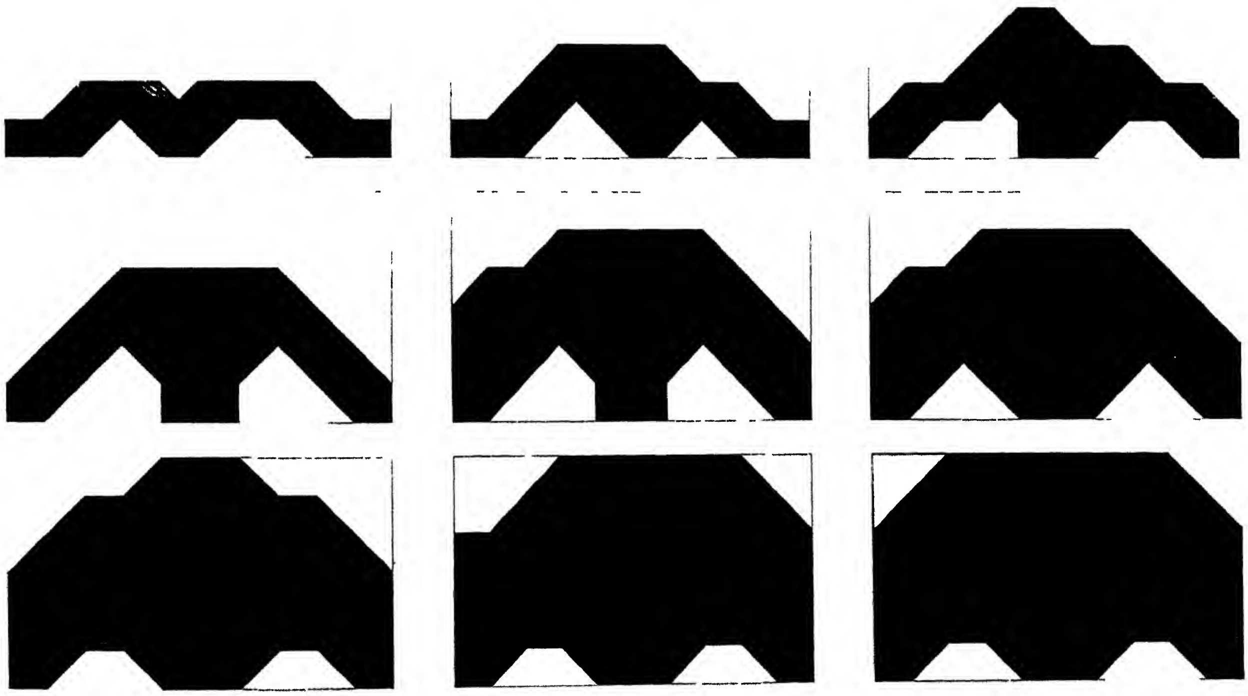

Figure 5 18 Nine trade-off shapes for the design of simple supported plate with a point load applied on the bottom side when weigt of the design is also considered

5.6 Design of the simple supported plate with a distributed load applied on the bottom

The loading and support conditions for this problem is shown in Figure 3 5 The loading is applied such that the all elements in the last row must be present to make the solution feasible The solution for this problem can eliminate the rows from the top if needed, without making the solution geometrically infeasible

5 6 1 Weight of the plate is not considered

First the problem is solved for the case when the weight of the designed plate is not taken, while calculating the stresses The result for such case is presented in Figure 5 19 The solution set obtained from the NSGA-II runs is fairly diverse and the shape of the Pareto-optimal front is also nearly smooth The deflection for this problem is found to be more than other problems These solutions undergo hill climbing local search and the solution set is further improved The solutions after the local search have got the ends of the Pareto-optimal front stretched by a small amount The non-dominated set of 25 solutions forms a perfect Pareto-optimal front Effective number of the different solutions is lesser than the non-dominated solutions as many solutions from the NSGA-II runs have converged to one solution Clustering helps to eliminate the solutions which are very near in the objective space or are co-incident Set of nine different solutions found by clustering is presented in the Figure 5 20

The solution set obtained is very interesting The solution with the minimum weight have the last row only Rest whole of the plate is eliminated This solution is the same as found for the single objective case solution while optimizing for the minimum weight The second solution, element (1,2) of the 3×3 matrix, has more weight placed symmetrically This follows the shape of the bending moment diagram to have an almost uniform distribution of the stresses This solution places the material in a compact manner such that the deflection is reduced substantially Next solution (1,3) also exhibits the same trend But the improvement in the weight and deflection is not as much as it has been in the second solution Solution (2,1) and (2,3) are having a cavity in the middle This helps in reducing the weight without affecting the deflection much The intermediate solution is also present in fifth solution (2,2) The solution (3,1), (3,2) and (3,3) exhibit much difference in the weights but the difference in deflections is not much This shows that putting very much weight in the solution will not help much to reduce the deflection beyond a limit The solution with the maximum weight is having the complete plate as the solution In this design, almost all the solutions are symmetric and the spread obtained is also very good

5 6 2 Weight of the plate is also considered

Now the weight of the design also increases the loading as the weight of the design increases The result for this case is presented in the Figure 5 21 The solution set of 30 non dominated individuals obtained by the NSGA-II simulation runs is worked on by the local search method The local search helps the solution to get a more stretched Pareto- optimal front But on the front many solutions are found to converge to one point This reduces the number of different solutions to 15 This is due to the fact that the weights are determined without taking diversity of solutions into account These non-dominated solutions form a smooth Pareto-optimal front The spread of the solutions is slightly different than the case when the weight is not considered This is reduced on the weight axis and more interestingly the range on the deflection axis is increased Nine of these solutions are taken and are presented in the Figure 5 22

The solution with the minimum weight is similar to the case, when weight of the plate is not considered This eliminates four rows from the top Infact, all the solutions in the first row of the 3×3 matrix are the same as the previous case when weight is not accounted for Solution (1,2) and (1 3) follow the same trend and have the symmetric solutions The second solution is such that there is much reduction in the deflection as compared to the weight as evident from the Figure 5 22 Solution (2 2) and solution (2 3) have got one hole This hole help in reducing the weight Though these solutions are not symmetric the hole causes the change in stresses more, as this is experiencing a unidirectional load More interestingly, the solutions (3,1) and (3,2) have more number of cavities It will be a matter of interest to find out the effect of the multiple holes on the solution apart from the reduction in the weight of the solution The solution with the maximum weight is having a cavity and also it has got some material cut on the corners, making the solution different from the maximum weight solution for the case when the weight is not considered

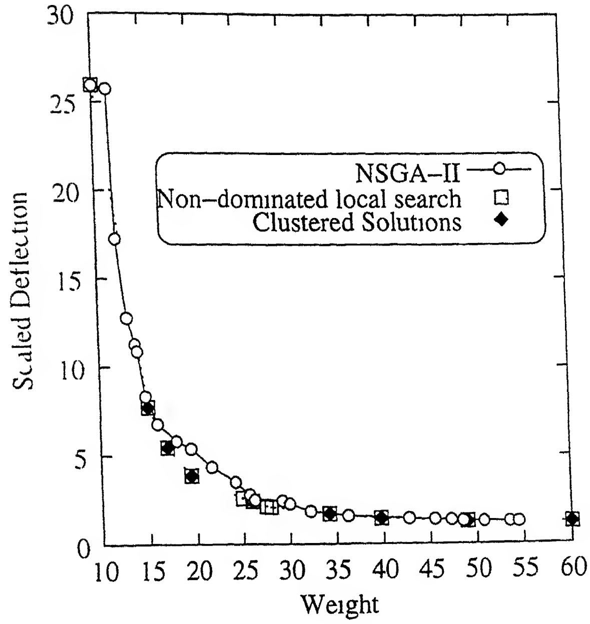

Figure 5 19 Hybrid procedure to find nine trade-off solutions for the simple supported plate design problem when the distributed load is applied on the bottom side and weight of the design is not considered

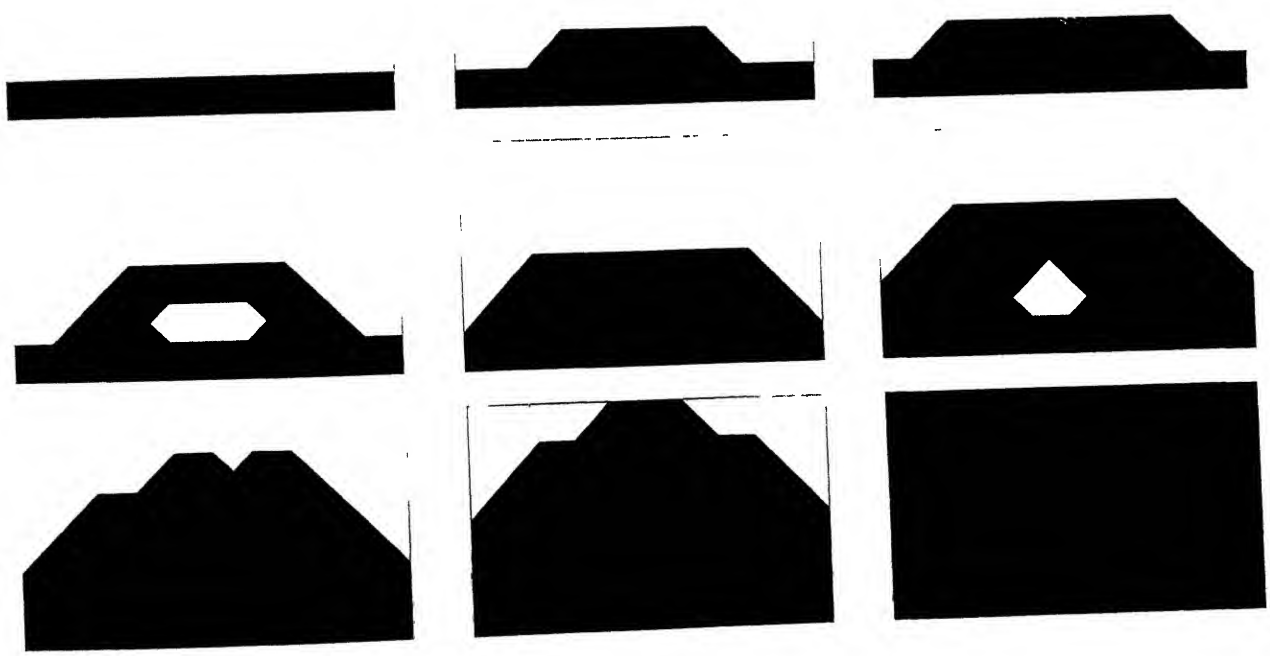

Figure 5 20 Nine trade-off shapes for the simple supported plate design when a distributed load is applied on the bottom side

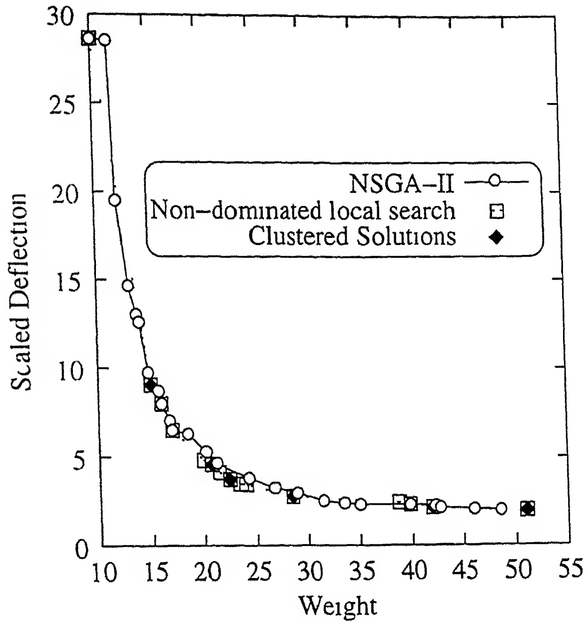

Figure 5 21 Hybrid procedure to find nine trade-off solutions for the simple supported plate design problem with a distributed load applied on the bottom side and weight is considered

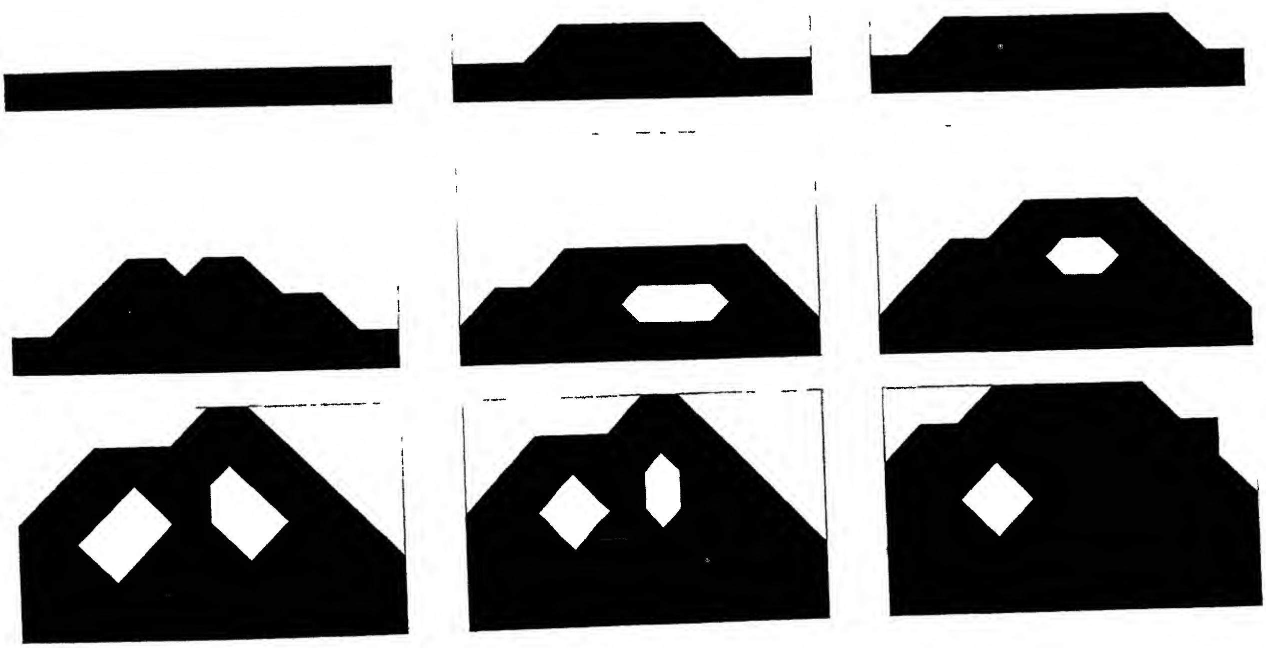

Figure 5 22 Nine trade-off shapes for the simple supported plate design when a distributed load is applied on the bottom side

5.7 Design of the hoister plate.

The loading and the support conditions for this problem are shown in the Figure 3 6 The size of the plate taken is 80×60 mm^2 This is a tough problem so some problem information regarding the cavity was necessary and hence specified in the problem This is the problem, where the strategy of having fixed weights has not been found working, hence the strategy of continuously updated weight is applied here and found to work well The solution for both the cases when the weight of the design is considered and when the weight is not considered are presented here

5 7 1 When weight is not considered

Figure 5 23 presents set of 30 non dominated solutions obtained by the NSGA-II runs The local search on this set of solutions gives 21 non dominated solutions The solution set after the local search is reduced to a small number and further more the number of different solutions is lesser than this number The solutions are such that there is a sharp bend in the Pareto-optimal front This solution set is reduced to nine solutions which are different in objective space as well as are diverse The different trade-off solutions are shown in Figure 5 24

This is a very very tough problem as we are trying to get a shape with a structure open at one end which give rise to unbalanced moments, thus causing higher stresses and deflection The solution set obtained finally shows the ability of the method to solve this problem also The solution with the minimum weight (solution (1,1) of the matrix) obtains the shape of a hook, very often used for the purpose of the hoister Most interesting thing about this solution is the opening on the left hand side without specifying any problem information of the same It is also interesting to note here that the solution for the minimum weight does not have the leftmost and rightmost columns and bottom most row Second solution is also interesting as this reduces the weight by a small amount but the change in the deflection is very much This is due to the change in the configuration The new shape is like the gripper of some arm This is connected to the upper side of the plate, hence there is not so much unbalanced moment as is experienced in the first solution Third solution reduces any unbalance due to the loading by creating a symmetry of loading This solution further reduces the deflection without affecting the weight by a large amount Rest all other solutions do not experience big changes in deflection despite of large changes in the weight of the design Solution (3,1) is a symmetric solution This solution smartly eliminates the part of the left and right columns and also the corners on the bottom are chopped The solution with maximum weight takes the maximum amount of plate that has been allowed The last three solutions have got a big change in the weights but the change in the deflection is not too much, showing the places where the weight is present without contributing much to the problem This information helps in optimizing the solutions for unknown cases

5 7 2 Design of the hoister when its own weight is also considered

The solution set of 30 non dominated solution obtained by NSGA-II runs is presented in Figure 5 25 The Pareto-optimal front has a step Interestingly, only 22 of these non dominated solutions are different But the diversity on the front is pretty well The local search gives 23 non dominated solutions out of which there are only 11 distinct solutions Nine solutions are taken out of the local set and are presented in the Figure 5 26

The solution set has got interesting shapes The solution set with the minimum weight is having an opening on the left hand side This has the arm joining the load carrying part to the support This makes the solution of minimum weight The weight of this solution is even lesser than the minimum weight solution of the previous case The solution (1,2) is also interesting as there is some reduction in the deflection with the weight increase Second and third solution make large compromise on the deflection axis to accommodate small increase in the weight The solution (2,1) gets a twist in the arm joining the loaded part and the support hence the deflection is reduced more causing a step in the Pareto-optimal front Other solutions do not have much difference in the deflection values with the weight change Interestingly, many of the solutions has eliminated the last row of the considered search space It is interesting to note that the solutions which puts material in the last row does not have much difference in the deflection values Thus last row material can be safely eliminated to reduce the weight, without effecting the deflection significantly (if required) The solution with the maximum weight is again interesting as this solution does not make use of whole plate, instead it eliminates the solutions on the left and right columns The solution is interesting as the increase in the weight is very much but the corresponding change in the deflection is not much for last five solutions Thus very steep slope with respect to one axis is obtained This solution is different than the previous case solution of maximum solution due to the reason that the weight is affecting the deflection also in this problem

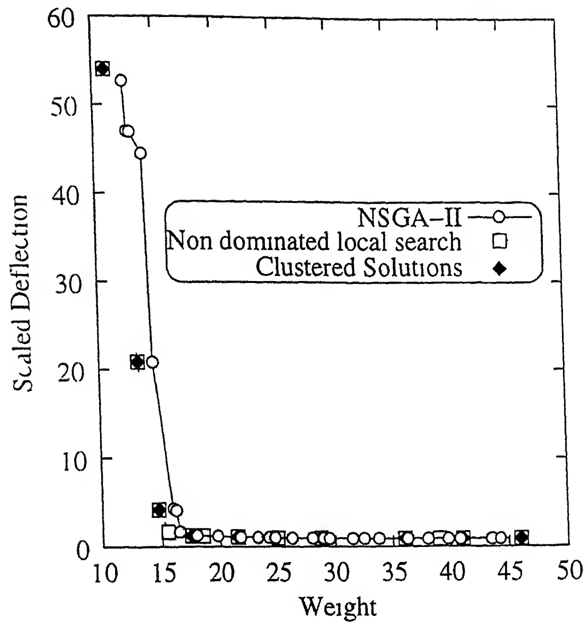

Figure 5 23 Hybrid procedure to find nine trade-off solutions for the hoister design problem when weight of the plate is not considered

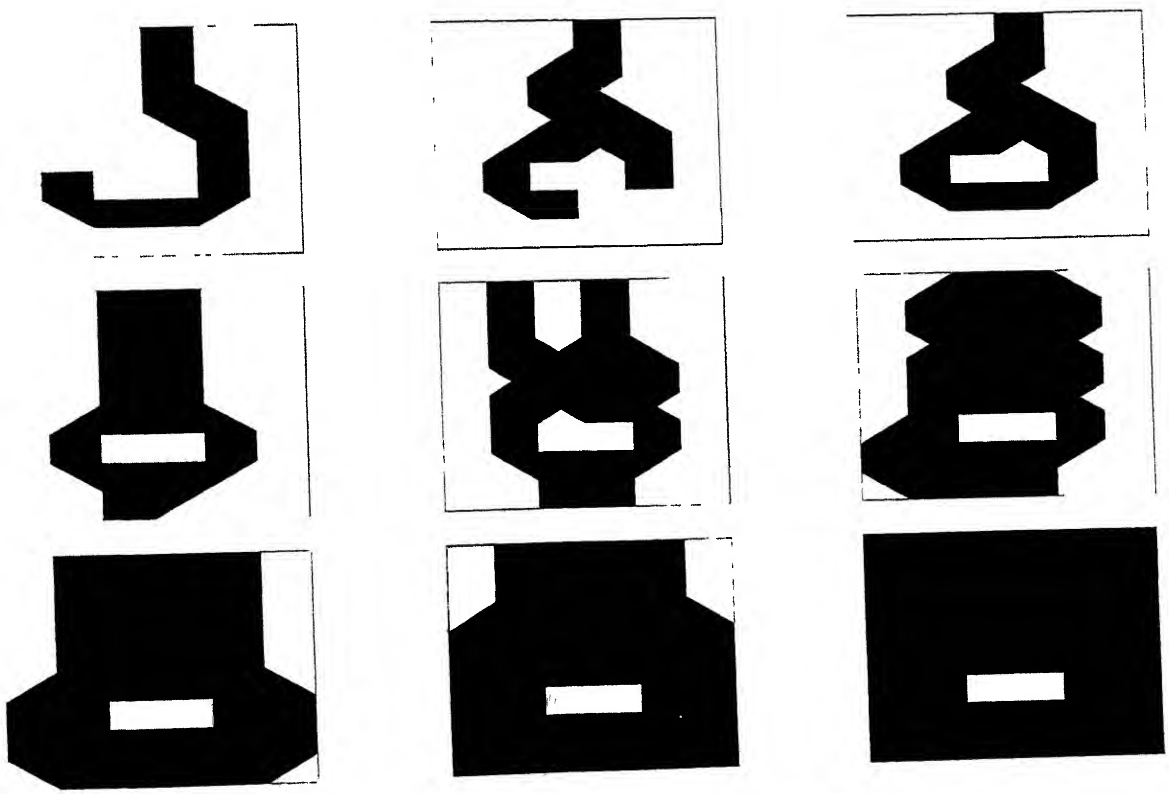

Figure 5 24 Nine trade-off shapes for the hoister plate design

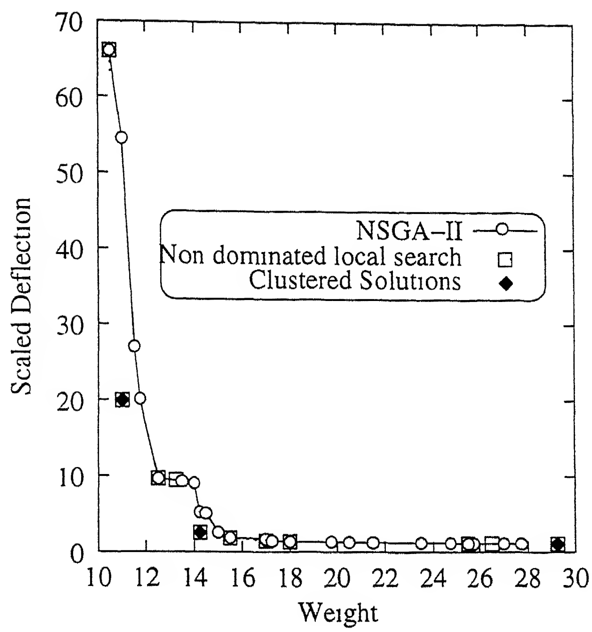

Figure 5 25 Hybrid procedure to find nine trade-off solutions for the hoister design problem when weight is also considered

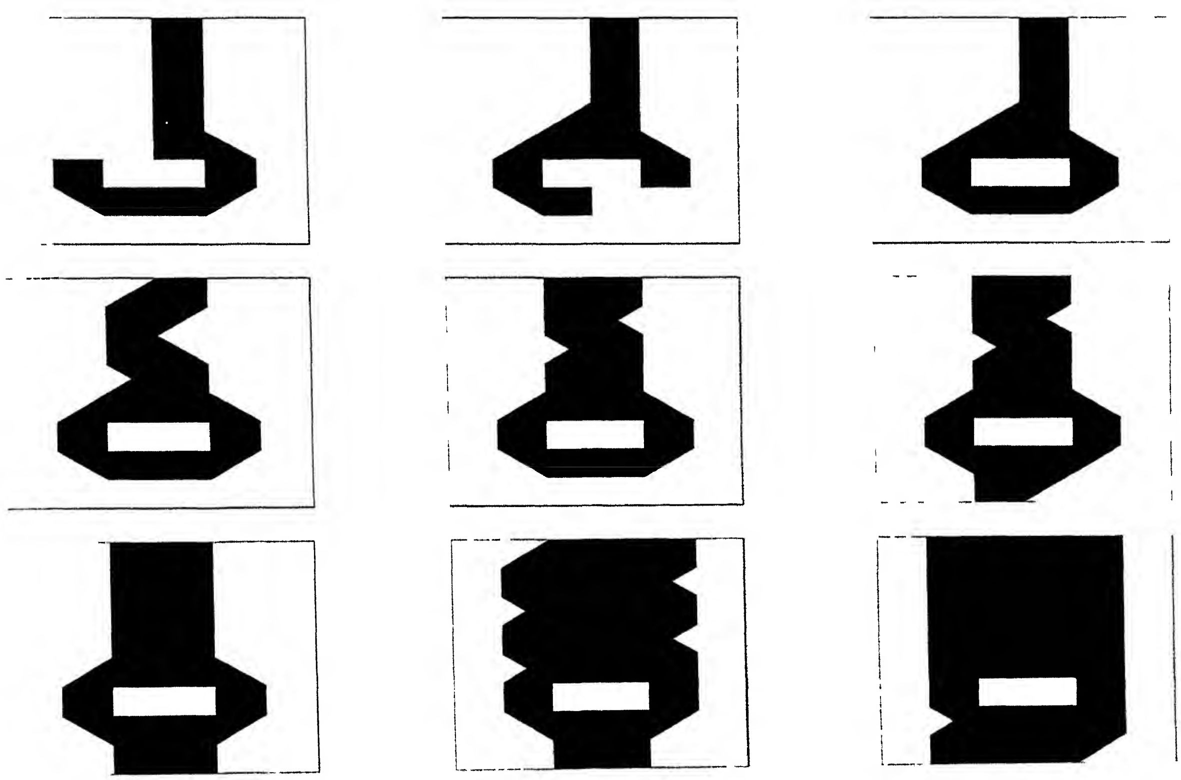

Figure 5 26 Nine trade-off shapes for the hoister design when weight is considered

5.8 Design of the bicycle frame.

Figure 3 7 shows the loading and the supports in the frame under consideration This problem is very interesting as direct application of this work is presented in the real world problems The design of frame is made with a very high static load than usually experienced The hybrid approach of the design gives a number of solution which gives not only optimal shapes but also the innovative ideas for the different shapes of the bicycle

5 8 1 When weight of the bicycle frame is not considered

Figure 5 27 present the results of the NSGA-II simulation runs The diversity of the solutions on the Pareto-optimal front is very good The non dominated solutions obtained after the local search form a new refined Pareto-optimal front The obtained Pareto-optimal front clearly resembles the initial idea described The spread of the solutions on the new obtained Pareto- optimal front is good, and the range of fitness values is improved Out of 25 non-dominated solutions there are 20 different solutions Five solutions from this widely spread front are selected and presented in the Figure 5 28 Lesser solutions are considered here, to avoid the cluster of solutions available for the designer, which make the task of decision making difficult

The shape of the bicycle frames obtained by the hybrid approach are presented in the form of the bicycles The shapes obtained are very interesting The solution for the minimum weight is very interesting as this has a vertical bar joining the seat with the paddle bracket and the rear wheel such as maximum load is taken by this part of the design The seat is joined to the handle by another bar This design is not available in the market, but it can be a good solution to consider Next shape obtained is a derivative of the minimum weight solution This solution thickens the vertical bar joining the seat and the rear wheel and the paddle bracket Slight modification in the weight reduces the deflection by a large amount It is interesting to see how the hybrid approach eliminates the material to find optimal solutions Another solution has got a slanted bar joining the handle and the paddle support This solution is similar to the most common cycle available in the market The thickening of the arm joining the rear wheel and the seat make it stiffer by reducing the stresses and hence strains Most interesting thing about this solution is the cavity obtained by the solution The solution with the maximum weight eliminate all the cavities The steps are found near to the front wheel This is due to modelling of the front wheel space as a staircase This solution has eliminated all the material in the last row but due to the requirement of paddle wheel bracket, one element is present in the last row

5 8 2 Design of the bicycle frame when its own weight is also considered

This design is more near to the real life design The Figure 5 29 presents the solutions obtained by the NSGA-II simulation runs This gives 25 different shapes of the bicycle frames The solution set undergoes the local search and 15 non-dominated solutions are obtained Local search increases the range of the solutions obtained on the weight axis but the range is considerably reduced on the deflection axis as is evident from the Figure 5 29 Most interestingly the number of distinct solutions is 8 The small number of solutions obtained show the efficiency of this approach for reducing the cardinality of this problem Out of these solutions five distinct solutions are selected using the clustering approach Consideration of the weight has got a significant increase in the deflection as compared with the case when the weight of the frame is not accounted These solutions are presented in the Figure 5 30

Here the shapes obtained are very interesting as it gives multiple solutions in one simulation run including some familiar shapes The solution obtained for the minimum weight has got a vertical arm joining the seat with the paddle support This paddle block is joined to the handle support and the rear wheel bracket The similar solution is seen in the day to day life The next solution is one step ahead of the minimum weight solution in the sense that this thickens the joint of the vertical arm joining paddle block and the paddle wheel with the rear wheel bracket Thus the solution becomes more stiff The solutions shown in the second row are also interesting This has a huge cavity in the solution without giving any problem information to the solution The solution is similar to one of the models available in the market, but it eliminates the bar joining the paddle block and the seat This shows that this arm can be eliminated to reduce the weight The next solution has put more material on the support The direct connection between the seat and the bracket for placing the paddle is not present but it is connected through the rear wheel bracket This arrangement reduces the stresses in this part of the design The maximum weight solution is not having whole of the plate, instead it has placed the material more on the joint of the rear wheel bracket and the seat and the handle joint with the seat, to reduce the stresses

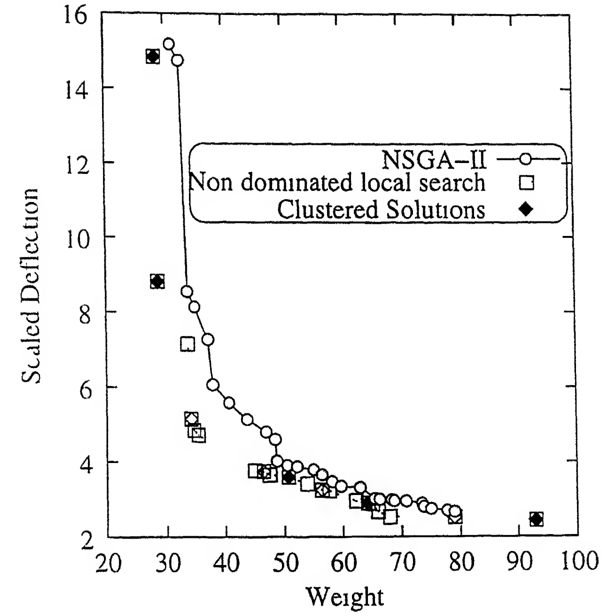

Figure 5 27 Hybrid procedure to find five trade-off solutions for the bicycle frame design problem when weight is not considered

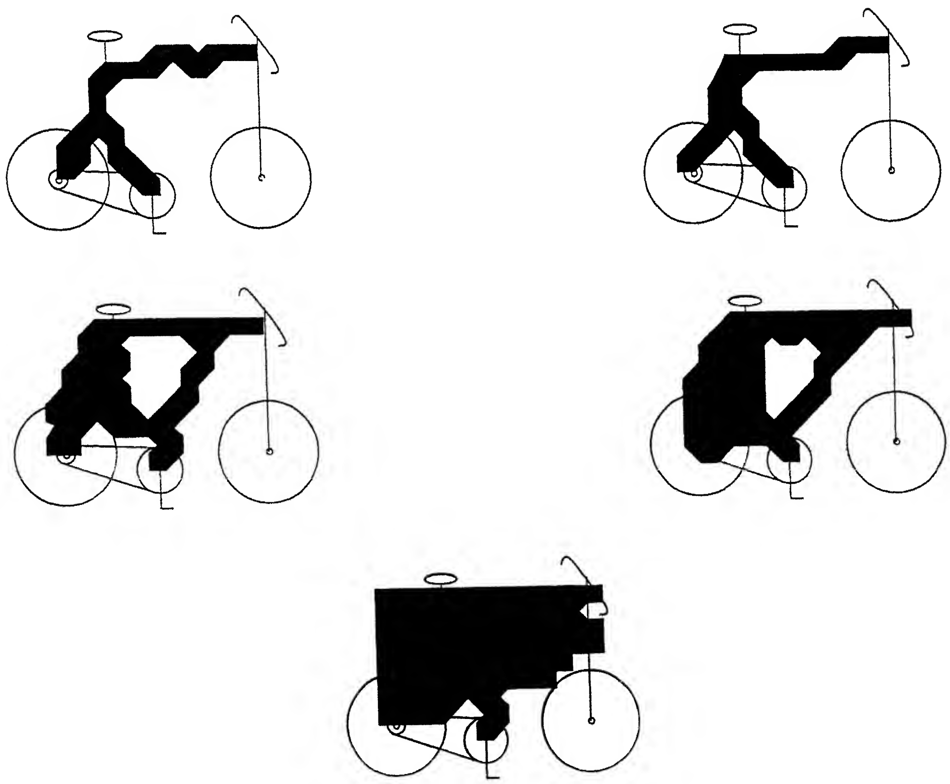

Figure 5 28 Five different designs of the bicycle frames when the weight of the frame is not accounted for the loading

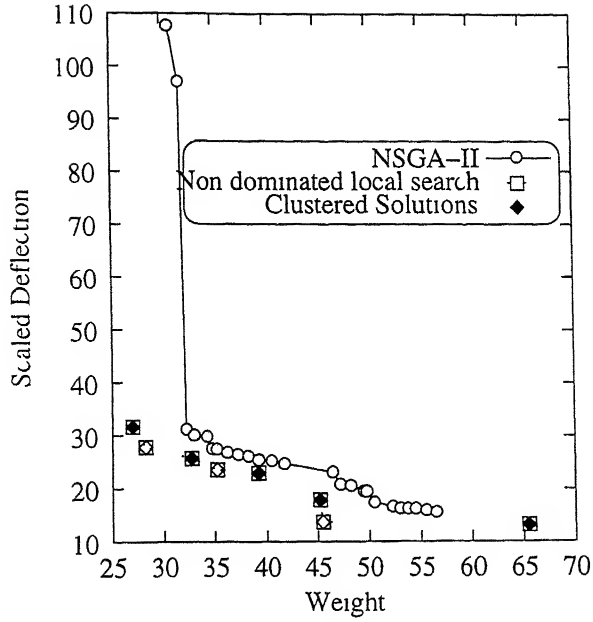

Figure 5 29 Hybrid procedure to find nine trade-off solutions for the bicycle frame design problem when weight is considered

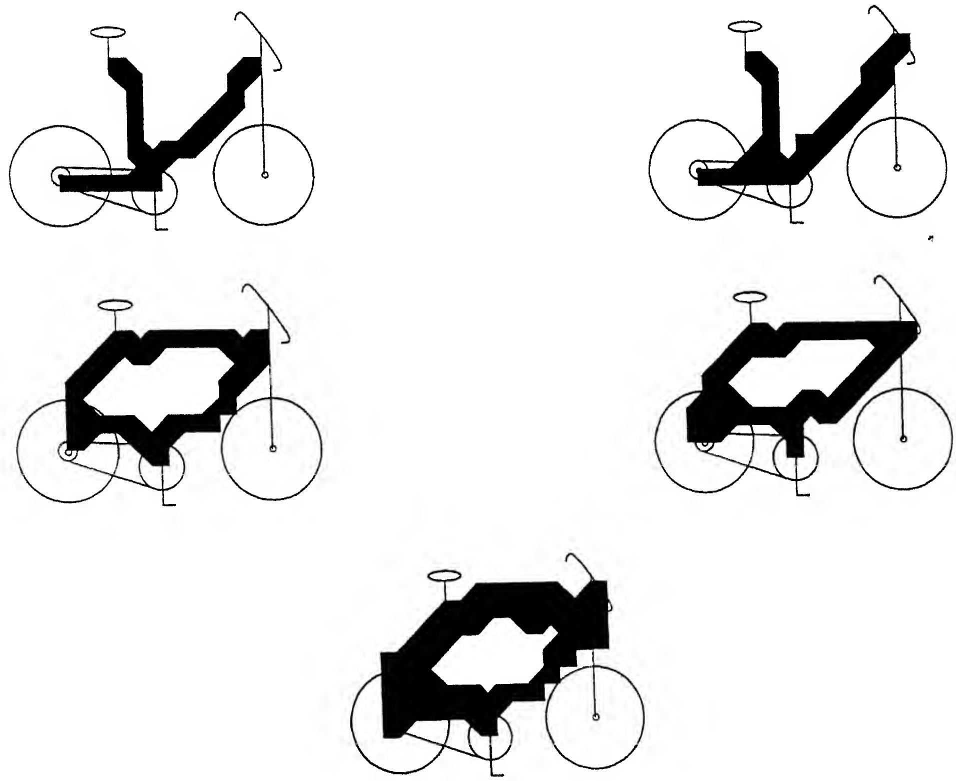

Figure 5 30 Five different designs of the bicycle frames when weight of the bicycle frame is also considered

5.9 Closure

The chapter presents the results for the different multi-objective optimization test problems for the cases when the weight of the designed plate is considered while calculating the weight and when weight is not applied to find the loadings The different problems are solved like—design of cross-section design of cantilever plate design of simple supported plate with a number of loading cases, design of the hoister plate The design of the bicycle frame is carried for a very high static load The hybrid method is used for solving different problems The hybrid method uses combination of NSGA-II and hill climbing local search

The results obtained by the study are very interesting In all the design problems two conflicting weights are used One is the minimization of the weight and other is the minimization of the deflection Varying results from solutions ranging from the minimum weight(maximum deflection) solution to maximum weight (minimum deflection) solution, are obtained The results obtained give the designer innovative ideas for the design of new shapes which have not been conceived so far A number of solutions are made available to the designer in a single simulation run The results for the real life problems like the design of the bicycle confirms the efficacy of the approach This makes the approach more attractive and feasible to solve the real world design problems of varying complexity

Chapter 6

Conclusion and Scope for future work

The present work proposes a new scheme of solving the shape optimization problems The design of the shapes is carried out by considering a rectangular plate as the basic shape for the design Here the shapes are represented by the presence or absence of small connected elements, which are represented by the binary numbers Thus the shape of any arbitrary structure can be represented by means of a string of binary numbers The shape obtained may have some disconnected regions To eliminate the disconnected region the biggest cluster of the connected elements is found The shape represented by this method may have some sharp corners To avoid that a smoothing method is devised and applied here to give the shape corresponding to the binary string

The binary GA can handle the binary strings so this is chosen as the method of solving the shape design problems Initially, a population of individuals is created randomly The binary individuals are converted into the shapes according to the above described method The shapes which satisfy the geometry constraints undergo finite element analysis, which give the maximum stress and maximum strain values The solutions which satisfy the stress and strain constraints are considered feasible The weight of these shapes is evaluated The solutions which are geometrically infeasible are assigned a very heavy penalty The shapes those are infeasible due to the stress or strain constraint violation are assigned an error value according to the violation of the constraint

For single objective cases weight is taken as the fitness function and design is carried out for the minimization of the weight Now standard GA operators i e selection, crossover and mutation are used to find new population of the individuals Here tournament selection with a constraint handling strategy is used for the selection procedure An innovative two dimensional crossover which respects the geometry of the individuals is devised and used This crossover is found to be efficient in these problems Single point mutation is used for mutation The solution is evolved through GA and get to a good solution from where, the local search is carried out on this best individual found by the GA

When the problem is analysed for more than one conflicting objectives, the single objective GA is not suited as there are many optimal solutions The solution for these problems is a Pareto-optimal

set of solutions A specific multi-objective genetic algorithm —elitist non dominated genetic algorithm (NSGA-II) is used to solve the multi-objective problems This algorithm has the capability of reaching the global Pareto-optimal front as well as obtaining the diversity on the front Here two objectives are minimization of the weight and the minimization of the deflection There are constraints on the maximum deflection value and maximum stress values The population undergoes the selection crossover and mutation Elitism is used to preserve the best non dominated solutions found so far The Goldberg s ranking scheme with constrained dominance is used to rank the population The selection is done on the basis of the rank and the diversity on the Pareto-optimal front The two dimensional crossover and simple mutation is used Each solution of the best solution set obtained by the NSGA-II undergoes the local search This results in a set of solutions, which may not be non-dominated Non-dominated solutions are identified by a non-domination search on this set of solutions From this final set of non dominated solutions, as many as required diverse solutions are selected using the clustering approach

The local search is done by flipping one bit at one time in an ordered manner Every time the shapes are extracted from the new string This is analysed and if the new solution is better than the previous solutions, the change is accepted and if this is not, the change is rejected and the previous string is restored The local search technique uses a single objective strategy hence it poses no problem for the single objective problems For multi-objective problems a weighted sum strategy is used to convert the multiple objectives into single objective The weights are determined by using the fitness values of the solution Two strategies are used for determining weights, one of the constant weights strategy i e the weight vector is determined once, and second strategy is of continuous updating of the weights The optimum solution obtained after each local search is used to update the weight vector

6.1 Conclusion

A number of shape optimization problems are solved using the hybrid approach The problems considered are design of cross-section, cantilever plate, simple supported plate, hoister design are considered A problem of design of bicycle frame is also used to show the application of the approach to the real world problems The main results can be summarized as following -

- The proposed hybrid approach is found to be very efficient to solve the shape optimization problems
- The solution depends on the representation scheme Proper representation is required to have the accuracy of the solution
- The proposed elemental approach of representing shapes is helpful in the finite element analysis, which also break the search domain into smaller elements Thus one computational step is reduced

- The use of elemental approach make it easier to find the solution for the cases when the properties of the material are varying over the search space

- For the real world problems, where the optimal solution of the problem is not known it is safe to get near to optimal solution and then use of local search with a good solution to get the optimal solution This may prove an efficient approach to find the optimal solutions

- It is known that the classical local search methods has the potential to reach the optima if the initial guess is very good The same knowledge is used in this hybrid method to ensure the convergence to the global optimal solution The genetic algorithm takes the solution to the basin of global optima and then local search starting from a good initial solution can take the solution to optima

- Since the local search method reaches the optima very efficiently if it starts from a good solution so a considerable amount of computational effort can be reduced by using the proposed approach

- It is found that since the solutions are only governed by the fitness values, many configurations can be obtained for the same fitness

- The proposed method has the potential of finding the shapes which are obvious and results difficult to visualize can also be obtained without any difficulty

- The proposed method can be very helpful in finding the new innovative solutions for the problem which may prove useful in conceptualizing new designs This feature of the considered approach make it extremely useful for the designer

- This approach follows the evolution process to find the best possible shape thus mimicking the nature, hence the solution found are the best possible cases in most of the cases making the best use of the resources Hence this approach can be of interest in the wake of globalization, as the job of designer to make the best possible use of the resources is easily achieved

- The approach presented here give a more practical look towards the use of genetic algorithms particularly the multi-objective genetic algorithms in practice to solve the real world problems

- For the multiple objective solutions it is possible to get the multiple solutions in one simulation run

- For the multi-objective cases the set of Pareto-optimal solution is obtained by the NSGA-II This is further improved by the local search and hence convergence to the global Pareto-optimal front is ensured

1. The approach proposed here shows a way of using the genetic algorithms for the real world problems. It's efficiency is well established through a number of test problems. This call for immediate use of the proposed scheme to solve the real world optimization problems of structural optimization.

2. The approach can be extended to the 3-dimensional cases by making use of three dimensional elements.

3. The proposed approach uses a small number of elements to represent the shapes. It will be interesting to find the effect of increasing dimensionality of the problem.

4. The proposed approach uses first order polynomial for smoothing purposes. It will be interesting to find the applicability of higher order polynomials for this purpose.

5. The optimal mesh generation techniques can be used to reduce the computational time.

6. One interesting task may be the parallelization of the process to make the method more practical by reducing the computational time.

7. The solution of the bicycle design can be physically formulated and verified for the applicability of the design.

8. The design of the fragmented design and then coupling of all the designs to get the assembly design may be an interesting work.

9. The method can be used to find the solutions when the properties of the material are not constant over the domain, instead they are varying.

10. The proposed approach can be used for different optimization problems from other fields as well.

6.3 Closure

The chapter presents the brief sketch of the methodology used for representing the shapes and then application of hybrid approach to solve the problems of shape optimization. The results obtained from the different test problems solved are used to draw certain conclusions. The main points are presented in the conclusions. The results show the applicability of the approach to the real world problems of shape design. This approach can be used to solve the real world problems of the fields other than structural optimization. The approach can also be extended to a more realistic case of 3-dimensional analysis.

The future scope of the work includes the applicability of the considered approach to the problems in other walks of life and also it asks for the physical manufacturing of the bicycle frame. The extension

to 3-dimensional cases and the use of fragmented design approach are also interesting fields for the future works. Use of parallel GA to find the solution of these problems will lead to the reduction in the computational time. This is required to be proved by the use of parallel structure of GA.

Bibliography

[1] Belegundu A and Chandrupatla, T R , *Introduction to Finite Elements in Engineering* Second Edition Prentice-Hall,India

[2] Braibant V and Fleury, C (1984) Shape optimal design using B-splines *Computer Methods in Applied Mechanics and Engineering*, 44, 247-267

[3] Chankong V and Haimes, Y Y (1983) *Multi-objective decision making theory and methodology* New York North-Holland

[4] Chapman C D , Jakiela, M J (1996) Genetic algorithms based structural topology design with compliance and topology simplification considerations *ASME Journal of Mechanical Design, 118* 89 98

[5] Chapman C D , Saitou, K , Jakiela, M J (1994) Genetic algorithms as an approach to configuration and topology design *ASME Journal of Mechanical Design, 116*, 1005–1012

[6] Deb K Pratap, A , Agrawal, S and Meyarivan, T (2000) A fast and elitist multi-objective genetic algorithm NSGA-II Technical Report No 2000001 Kanpur Indian Institute of Technology Kanpur India

[7] Deb K , Agrawal, S , Pratap, A , Meyarivan, T (2000) A Fast Elitist Non-dominated sorting genetic algorithm for multi-objective optimization NSGA-II *Proceedings of the Parallel Problem Solving from Nature VI Conference*, pp 849–858

[8] Deb K (1999) Multi-objective genetic algorithms Problem difficulties and construction of test Functions *Evolutionary Computation, 7*(3), 205–230

[9] Deb , K (2000) An efficient constraint handling method for genetic algorithms *Computer Methods in applied mechanics and engineering*, 186, 311-338

[10] Deb, K (2000) Non-linear goal programming using multi-objective genetic algorithms *Journal of Operational Research Society*, 51 (in press)

[11] Ding, Y (1986) Shape optimization of structures–A literature survey *Computers and Structures* 24(6), 985-1004

[12] Ding Y (1987) Shape optimization of two-dimensional elastic structures with optimal thickness for fixed parts *Computers and Structures*, 27(6), 729-743

[13] Duda J W and Jakiela M J (1997) Generation and classification of structural topologies with genetic algorithm speciation *ASME Journal of Mechanical Design*, *119*, 127–131

[14] Espiga F Gracia L and Doblare, M (1989) Shape optimization of elastic homogeneous 2-D bodies by the boundary element method *Computers and Structures*, 33(5), 1233-1241

[15] Fonseca C M and Fleming, P J (1993) Genetic algorithms for multi-objective optimization Formulation, discussion and generalization *Proceedings of the Fifth International Conference on Genetic Algorithms* 416–423

[16] Ghoshtasby, A (1995) Geometric modelling using rational Gaussian curves and surfaces *Computer Aided Design*, 27(5) 363-375

[17] Goldberg D E Deb K, and Clark, J H (1992) Genetic algorithms, noise, and the sizing of populations *Complex Systems, 6*, 333–362

[18] Haftka, R T and Grandhi, R V (1986) Structural shape optimization–A survey *Computer methods in Applied Mechanics and Engineering*, 57, 91-106

[19] Hamada H and Schoenauer, M (2000) Adaptive techniques for evolutionary optimum design *Proceedings of the Evolutionary Design and Manufacture* , pp 123–136

[20] Holland, J H (1975) *Adaptation in Natural and Artificial Systems* Michigan MIT Press

[21] Horn, J and Nafploitis, N , and Goldberg, D E (1994) A niched Pareto genetic algorithm for multi-objective optimization *Proceedings of the First IEEE Conference on Evolutionary Computation* 82–87

[22] Jakiela, M J , Chapman, C , Duda, J , Adewuya, A , and Saitou, K (2000) Continuum structural topology design with genetic algorithms *Computer Methods in Applied Mechanics and Engineering, 186*, 339–356

[23] Kim, H , Querin, O M , and Steven, G P (2000) Post-processing of the two-dimensional evolutionary structural optimization topologies In I Parmee (Ed) *Evolutionary Design and Manufacture*, London Springer pp 33–44

[24] Knowles J and Corne, D (1999) The Pareto archived evolution strategy A new baseline algorithm for multi-objective optimization *Proceedings of the 1999 Congress on Evolutionary Computation* Piscataway New Jersey IEEE Service Center, 98–105

[25] Liu B Haftka R T Akgun, M A , Todoroki, A (2000) Permutation genetic algorithm for stacking sequence design of composite laminates *Computer Methods in applied mechanics and engineering* 186 (2-4) 357 372

[26] Rade L Westergren B *Mathematics Handbook for Science and Engineering* 4^{th} Edition Berlin Springer

[27] Reddy J N *An Introduction To The Finite Element Method* , $2^{n}d$ Edition, McGraw-Hill Inc

[28] Sandgren E Jensen, E and Welton, J (1990) Topological design of structural components using genetic optimization methods *Proceedings of the Winter Annual Meeting of the American Society of Mechanical Engineers*, pp 31–43

[29] Schaffer J D (1985) Multiple objective optimization with vector evaluated genetic algorithms In J J Grefenstette(Ed), *Proceedings of an International Conference on Genetic Algorithms and Their Applications*, Pittsburgh, PA, 93-100

[30] Sen P and Yang, J -B (1998) *Multiple criteria decision support in engineering design* London Springer

[31] Shyy Y K , Fleury, C , and Izadomanah, K (1988) Shape optimal design using higher order elements *Computer Methods in Applied Mechanics and Engineering*, 71, 99-116

[32] Srinivas N and Deb, K (1995) Multi-Objective function optimization using non-dominated sorting genetic algorithms *Evolutionary Computation*(2), 221–248

[33] Tavakkoli S and Dhande, S G (1991) Shape synthesis and optimization using intrinsic geometry *ASME Journal of Mechanical Design*, 113(4), 379-386

[34] Timoshenko, S P and Goodier, J N *Theory of Elasticity*, McGraw-Hill, New York, 1970

[35] Venktesh J (1994) Shape Optimization using intrinsic geometry and boundary element method *Doctoral Dissertion*, Department of Mechanical Engineering, IIT-Kanpur

[36] Zitzler, E (1999) Evolutionary algorithms for multi-objective optimization Methods and applications *Doctoral thesis ETH NO 13398*, Zurich Swiss Federal Institute of Technology (ETH) Aachen Germany Shaker Verlag

[37] Zitzler, E., Deb, K. and Thiele, L. (2000). Comparison of multi-objective evolutionary algorithms: Empirical results. *Evolutionary Computation Journal*, *8*(2), 173–196.

[38] Zitzler, E. and Thiele, L. (1998). An evolutionary algorithm for multi-objective optimization: The strength Pareto approach. *Technical Report No. 43 (May 1998)*. Zurich: Computer Engineering and Networks Laboratory, Switzerland.

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